AP

AP® Calculus AB
2014 Free-Response Questions

CollegeBoard

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

avg rate of change of $A(t)$ $= \dfrac{A(30) - A(0)}{30 - 0}$

$A(t) \rightarrow$ lbs
$t \rightarrow$ days

$= -.197$ lbs/day

1pt - answer w/units

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$A'(15) = -.164$ lbs/day

1pt - $A'(15)$

$f'(x) < 0$ means $f(x)$ dec

$A'(15) < 0$ means $A(15)$ dec

$A'(15)$ is amount of grass clippings @ $t = 15$ days is decreasing at rate of .164 lbs/day.

1pt - explanation

Continue problem 1 on page 5.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

$A(t)$ =

$\hookrightarrow \frac{1}{b-a}\int_a^b A(t)\,dt$

$A(t) = \frac{1}{30-0}\int_0^{30} A(t)\,dt$

$6.687(0.931)^t = \frac{1}{30}\int_0^{30} A(t)\,dt$

$t = 12.415$

1pt - $\frac{1}{30}\int_0^{30} A'(t)dt$

1pt - answer

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$y - y_1 = m(x - x_1)$

$L(t) - A(30) = A'(30)(t - 30)$

$L(t) = A'(30)(t-30) + A(30)$

2pts - $L(t)$

$0.5 = A'(30)(t-30) + A(30)$

1pt - sub $L(t) = 0.5$

$t = 35.054$

1pt - answer

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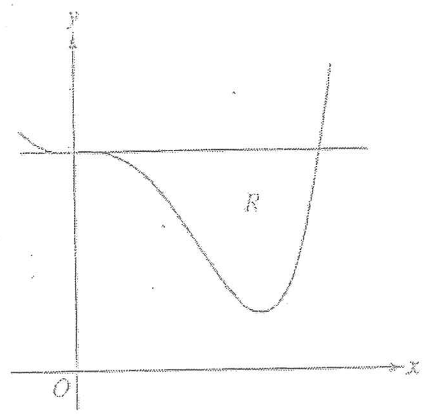
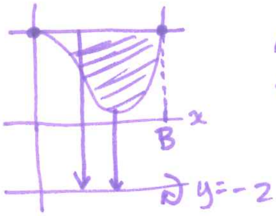


2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.



A = 0
B = 2.3

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$$V = \pi \int_A^B \left[(4 - -2)^2 + (f(x) - -2)^2\right] dx$$

$= 98.868$

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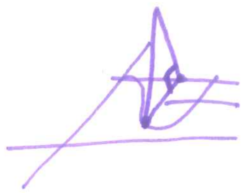
Continue problem 2 on page 7.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

$A = \frac{1}{2} bh$ (isosceles, $\therefore b = h$)

$= \frac{1}{2} b \cdot b$

$= \frac{1}{2} b^2$



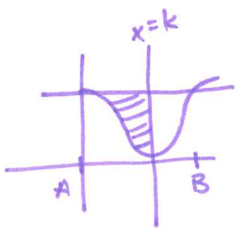
base of Δ $= 4 - f(x)$

$A = \frac{1}{2}(4 - f(x))^2$

$$V = \int_A^B \frac{1}{2}(4 - f(x))^2 \, dx$$

$V = 3.574$

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.



$$\frac{1}{2}(\text{Area of } R) = \int_A^k (4 - f(x)) \, dx$$

$$\frac{1}{2} \int_A^B (4 - f(x)) \, dx = \int_A^k (4 - f(x)) \, dx$$

or

$$\int_A^k (4 - f(x)) \, dx = \int_k^{2.3} (4 - f(x)) \, dx$$

... area from A to k = area from k to B

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