



AP[®] Calculus AB

2016 Free-Response Questions

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| t (hours) | 0 | 1 | 3 | 6 | 8 |
|---------------------------|------|------|-----|-----|-----|
| $R(t)$ (liters / hour) | 1340 | 1190 | 950 | 740 | 700 |

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

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- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

-
- (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

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2. For $t \geq 0$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

- (a) At time $t = 4$, is the particle speeding up or slowing down?

-
- (b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

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(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

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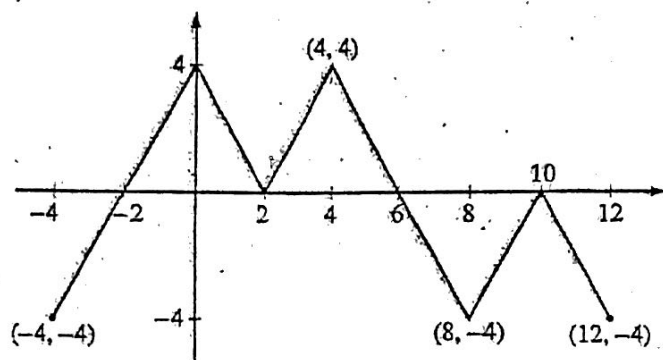
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Graph of f

3. The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_2^x f(t) dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

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- (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$.

Justify your answers.

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-
- (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

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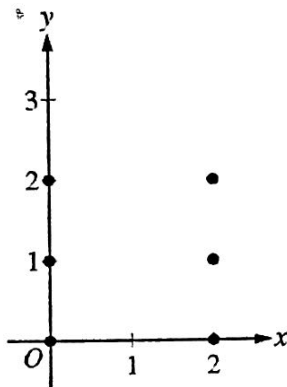
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4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



- (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

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(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

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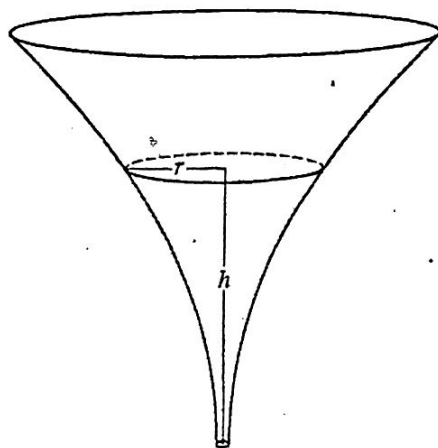
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5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.
- (a) Find the average value of the radius of the funnel.

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(b) Find the volume of the funnel.

-
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

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| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 1 | -6 | 3 | 2 | 8 |
| 2 | 2 | -2 | -3 | 0 |
| 3 | -8 | 7 | 6 | 2 |
| 6 | 4 | 5 | 3 | -1 |

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

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(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x) dx$.

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