2017

AP CollegeBoard

AP Calculus AB

Free-Response Questions

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h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

$\Delta h = 2$, $\Delta h = 3$, $\Delta h = 5$

$(ft^2)(ft) = ft^3$

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

$\int_0^{10} A(h)\,dh = 2(50.3) + 3(14.4) + 5(6.5)$ ft^3

← ok to stop here

Volume of the tank $= 176.3$ ft^3

1 pt - left + sum

1 pt - approx

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(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

The approx in part (a) is an overestimate of volume of tank b/c A is decreasing

1 pt - overestimate w/ reason

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Continue question 1 on page 5.

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(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

Area of cross section = $f(h)$

$$\text{Volume} = \int_0^{10} \frac{50.3}{e^{.2h} + h}\, dh$$

$= 10.325 \text{ ft}^3$

1pt - integral

1pt - answer

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

$h = 5$ ft

use $\int f(h) = \text{volume}$

$\frac{dV}{dt} = ?$

$\frac{dh}{dt} = .26$ ft/min

$$\text{Volume} = \int_0^h f(H)\, dH$$

$$\frac{dV}{dt} = f(h) \cdot \frac{dh}{dt}$$

2pts → $\frac{dV}{dt}$

$= f(5) \cdot (.26)$

$= 1.694 \text{ ft}^3/\text{min}$

1pt → answer

1pt → correct units in (a), (c) and (d)

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2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

rate remove → $f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right)$ for $0 < t \le 12$,

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

rate added → $g(t) = 3 + 2.4\ln\left(t^2 + 2t\right)$ for $3 < t \le 12$,

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

bananas removed = $\int_0^2 f(t)\,dt$

1pt - integral

$= 20.051$ lbs

1pt → answer

↖ units not required

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

☺

$f(t) \to$ lbs/hr

$f'(t) \to \frac{\text{lbs/hr}}{\text{hr}}$

$f'(7) = -8.120$ lbs/hr²

1pt - value

$f'(7)$ means rate of bananas being removed at $t = 7$ hrs is decreasing by 8.120 lbs/hrs²

1pt - meaning

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Continue question 2 on page 7.

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(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

lbs bananas = initial + $\int$ rate added − $\int$ rate removed

rate of # lbs bananas = rate added − rate removed

$B(t) = g(t) - f(t)$

1 pt - considers $g(5)$ & $f(5)$

$B(5) = g(5) - f(5)$

$= -2.263$

# of lbs of bananas is decreasing @ $t = 5$ b/c

$g(5) - f(5) < 0$

1 pt - answer w/ reason

(d) How many pounds of bananas are on the display table at time $t = 8$?

lbs bananas @ $t = 8$ = initial + $\int_3^8$ rate added − $\int_0^8$ rate removed

$= 50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt$

$= 23.347$

2 pts ⇒ integrals
1 pt ⇒ answer

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