rate enter

1. Fish enter a lake at a rate modeled by the function E given by $\boxed{E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)}$. Fish leave the lake at a rate modeled by the function L given by $\boxed{L(t) = 4 + 2^{0.1t^2}}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

(a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ? Give your answer to the nearest whole number.

$\int$ rate enter

fish enter lake $= \int_0^5 E(t)\,dt$ 1pt- integral

$= 153.478$

≈ 153 1pt- answer

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ?

$\frac{1}{b-a}\int_a^b$ rate leave

avg # leave lake $= \frac{1}{5-0}\int_0^5 L(t)\,dt$ 1pt- integral

$= 6.059$ 1pt- answer

Continue question 1 on page 5.

(c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

↳ abs max → crit #s & endpts into orginal equation

fish in lake = ∫(rate enter − rate leave)

$F(t) = \int (E(t) - L(t))\,dt$ ☺

rate of fish in lake → $F'(t) = E(t) - L(t)$ rate enter − rate leave

$E(t) - L(t) = 0$

$t = 6.204$

$F(6.204) = \int_0^{6.204} (E(t) - L(t))\,dt = 135.015$

$F(0) = 0$

$F(8) = \int_0^8 (E(t) - L(t))\,dt = 80.920$

@ $t = 6.204$, there is greatest # of fish in lake

1pt: $E(t) - L(t) = 0$

1pt: answer

1pt: justification

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

↳ rate inc → (rate)' > 0

rate dec → (rate)' < 0

rate fish in lake = $E(t) - L(t)$ ← need derivative of this rate

$E'(t) - L'(t)$

$E'(5) - L'(5) = -10.723$

Rate of change in # of fish is dec @ $t = 5$ b/c $E'(5) - L'(5) < 0$

1pt: considers $E'(5)$ and $L'(5)$

1pt: answer w/ reason

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$\Delta t = 0.3$ $\Delta t = 1.4$ $\Delta t = 1.1$

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	−29	55	48

2. The velocity of a particle, P, moving along the x-axis is given by the differentiable function v_P, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.

← initial position

(a) Justify why there must be at least one time t, for $0.3 \le t \le 2.8$, at which $v_P'(t)$, the acceleration of particle P, equals 0 meters per hour per hour.

MVT? IVT?

$$v'_p(t) = \frac{v_p(2.8) - v_p(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

1pt: $v_p(2.8) - v_p(0.3) = 0$

1pt: justification using MVT

v_p is cont. on $[0.3, 2.8]$ b/c v_p is diff'able

∴, by MVT, there is a value t on $(0.3, 2.8)$ s.t. $v'(t) = 0$

$\frac{1}{2}(b_1 + b_2)h$

(b) Use a $\boxed{\text{trapezoidal sum}}$ with the three subintervals $[0, 0.3]$, $[0.3, 1.7]$, and $[1.7, 2.8]$ to approximate the value of $\int_0^{2.8} v_P(t)\,dt$.

$$\int_0^{2.8} v_p(t)\,dt \approx \frac{1}{2}(55+0)(0.3) + \frac{1}{2}(55 + -29)(1.4) + \frac{1}{2}(55 + -29)(1.1)$$

1pt: answer using trap sum

Continue question 2 on page 7.

(c) A second particle, Q, also moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $\boxed{v_Q(t) = 45\sqrt{t}\cos\left(0.063t^2\right)}$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.

$\int |v(t)|\,dt$

$v_Q(t) \ge 60?$

graph $v_Q(t)$

$v_Q(t) \ge 60$ on $[1.866, 3.519]$

1pt: interval

use stored values!

distance traveled $= \int_{1.866}^{3.519} |v_Q(t)|\,dt$

$= 106.109$

1pt: definite integral

1pt: answer

(d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.

position P − position Q

$x_P(2.8) = \underbrace{x_P(0)}_{\text{init position}} + \int_0^{2.8} v_P(t)\,dt$

$= 0 + \int_0^{2.8} v_P(t)\,dt$

≈ 40.75

from part b

$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\,dt$

$= -90 + \int_0^{2.8} v_Q(t)\,dt$

$= 45.938$

1pt: $\int_0^{2.8} v_Q(t)\,dt$

1pt: position of Q

1pt: answer

$45.938 - 40.75 = 5.188$

Distance btn particles is 5.188 m