CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

   (a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

   (b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

   (c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

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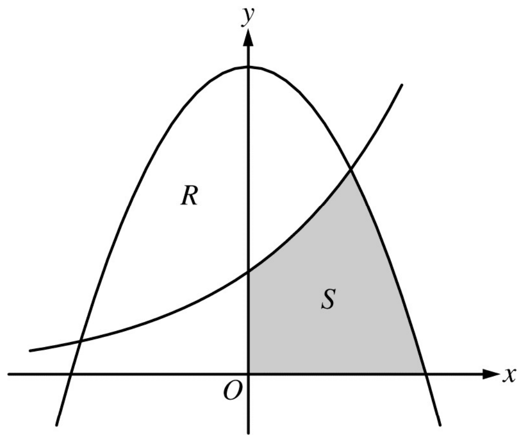

2. Let R and S in the figure above be defined as follows: R is the region in the first and second quadrants bounded by the graphs of $y = 3 - x^2$ and $y = 2^x$. S is the shaded region in the first quadrant bounded by the two graphs, the x-axis, and the y-axis.

   (a) Find the area of S.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with one leg across the base of the solid. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

3. The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

   (a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

   (c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

   (d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

END OF PART A OF SECTION II

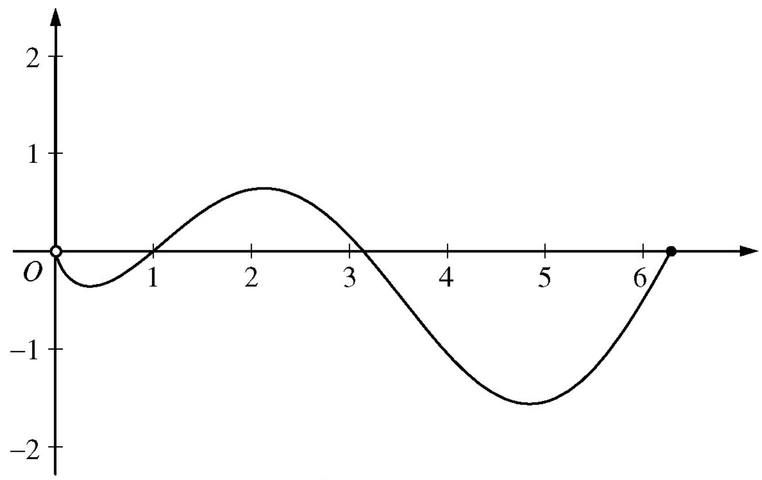
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

   (a) Find $g(1)$ and $g'(1)$.

   (b) On what intervals, if any, is g increasing? Justify your answer.

   (c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

   (d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

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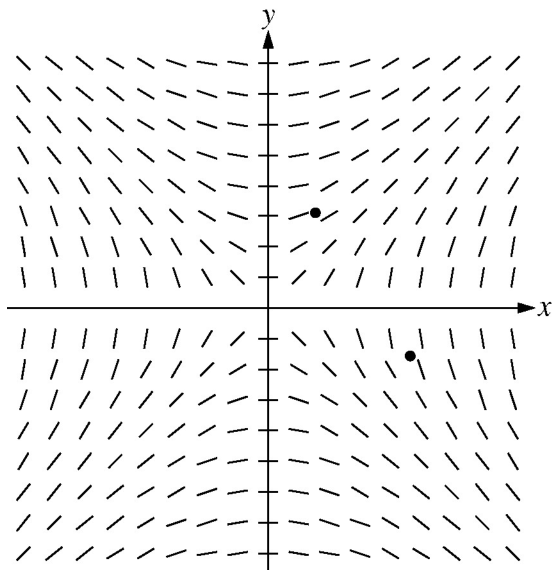
5. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x}{y}$, where $y \neq 0$.

(a) The slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(3, -1)$, and sketch the solution curve that passes through the point $(1, 2)$.

(Note: The points $(3, -1)$ and $(1, 2)$ are indicated in the figure.)



(b) Write an equation for the line tangent to the solution curve that passes through the point $(1, 2)$.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = -1$, and state its domain.

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6. Let $g(x) = xe^{-x} + be^{-x}$, where b is a positive constant.

(a) Find $\lim_{x\to\infty} g(x)$.

(b) For what positive value of b does g have an absolute maximum at $x = \dfrac{2}{3}$? Justify your answer.

(c) Find all values of b, if any, for which the graph of g has a point of inflection on the interval $0 < x < \infty$. Justify your answer.

STOP

END OF EXAM