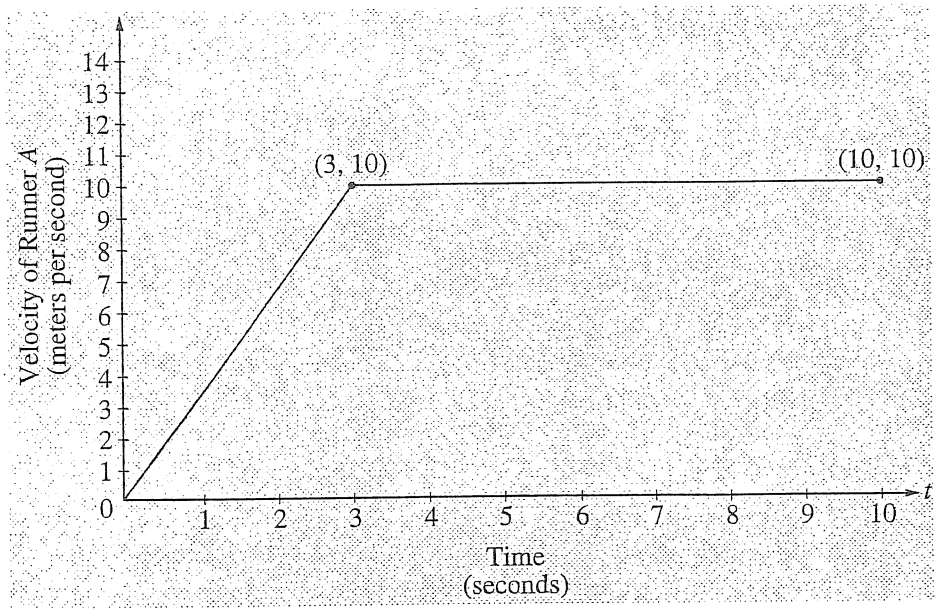
Topic 1: Particle Motion
Particle moving on a line

These questions may give the position equation, the velocity equation or the acceleration equation along with an initial condition. Students may be asked about the motion of the particle: its direction, when it changes direction, its maximum position in one direction (farthest left or right) etc. Speed, the absolute value of velocity, is also a common topic.

The particle may be a "particle," a person, car, etc. The position, velocity or acceleration may be given as an equation, a graph or a table. There are a lot of different things students may be asked to find. While particles don't really move in this way, the question is a versatile way to test a variety of calculus concepts.

What students should know how to do (AB and BC):

- Initial value differential equation problems: given the velocity or acceleration with initial condition(s) find the position or velocity.
- Distinguish between position at some time (displacement) and the total distance traveled during the time.
    - The total distance traveled is the definite integral of the absolute value of the rate of change (velocity): $\int_a^b |v(t)|\,dt$
    - The net distance (displacement) is the definite integral of the rate of change (velocity): $\int_a^b v(t)\,dt$
    - The final position is the initial position plus the definite integral of the rate of change from $x = a$ to $x = t$: $s(t) = s(a) + \int_a^t v(x)\,dx$ Notice that this is an accumulation function equation.
- Find the speed at a particular time. The speed is the absolute value of the velocity.
- Find average speed, velocity, or acceleration
- Determine if the speed is increasing or decreasing.
    - If at some time, the velocity and acceleration have the *same* sign then the speed is increasing.
    - If they have *different* signs the speed is decreasing.
    - If the velocity graph is moving away from (towards) the t-axis the speed is increasing (decreasing).
- Use a difference quotient to approximate derivative
- Riemann sum approximations
- Units of measure
- Interpret meaning of a definite integral in context of the problem



2. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t + 3}$.

(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

Continue problem 2 on page 7.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

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3. An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

Continue problem 3 on page 9.

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(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

3. A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

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Continue problem 3 on page 9.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

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A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

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(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \leq t \leq 2$.

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NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

   (a) Find the time t at which the particle is farthest to the left. Justify your answer.

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Continue problem 4 on page 11.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

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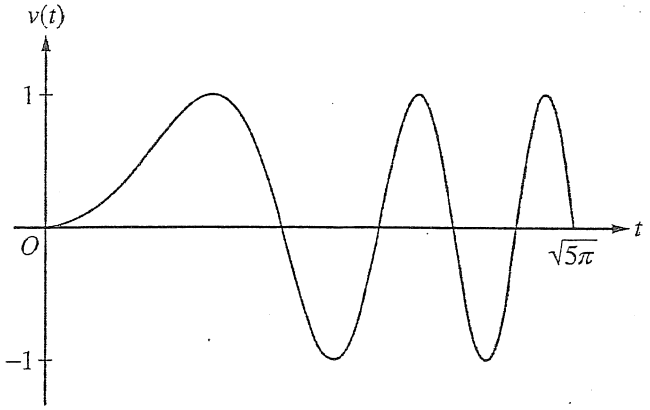

2. A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin\left(t^2\right)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

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Continue problem 2 on page 7.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \le t \le \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

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