Topic 2: Graphs of f, f', and f''
Interpreting from Graphs & Tables of Values

There are a variety of question types here. Students may be given an equation of the function or its derivative and asked for the location of extreme values, intervals where the function is increasing or decreasing, concavity, etc. Students may be given the graph of the derivative and asked the same kinds of things. They may be asked to find the value of the integral given the graph but no equation.

This may be a particle motion problem where the velocity is given as a graph.

What students should know how to do:

- Reading information about the function from the graph of the derivative. This may be approached as a derivative techniques or antiderivative techniques.
- Find and justify extreme values (1st DT, 2nd DT, Closed interval test (aka. Candidates' test).
- Find and justify points of inflection.
- Write an equation of tangent line
- Evaluate Riemann sums from graphs only.
- FTC: Evaluate integral from area of regions on the graph.
- FTC: Realize that if $g(x)=g(a)+\int_a^t f(t)\,dt$, then $g'(x)=f(x)$
- "Family of functions": functions with a parameter;
- Functions defined by other functions.

Tables may be used to test a variety of ideas in calculus including analysis of functions, accumulation, position-velocity-acceleration, *et al.*

What students should be able to do:
- Approximate the derivative using a difference quotient.
- Use Riemann sums (left, right, midpoint) or a trapezoidal approximation to approximate the value of a definite integral using values in the table (typically with uneven subintervals). (Trapezoidal Rule, *per se*, is not required.
- Average value and the MVT may appear
- Questions about the Rolle's theorem, MVT, IVT, etc.

Do not: Use a calculator to find a regression equation and then use that to answer parts of the question. (While finding them ids perfectly good mathematics, regression equations are not one of the four things students may do with their calculator and give only an approximation of our function.)

NO CALCULATOR ALLOWED

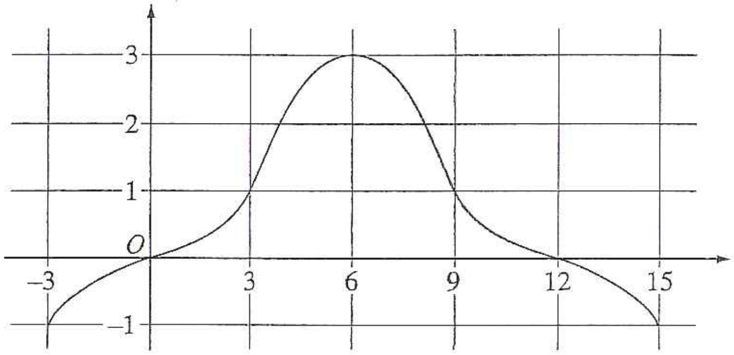
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

$g(6) = 5 + \int_6^6 f(t)\,dt$

$= 5 + 0$

$\boxed{g(6) = 5}$

$g'(x) = \frac{d}{dx}\left(5 + \int_6^x f(t)\,dt\right)$

$= f(x)$

$g'(6) = f(6)$

$\boxed{g'(6) = 3}$

$g''(x) = f'(x)$

$g''(6) = f'(6)$

$\boxed{g''(6) = 0}$

1pt : $g(6)$

1pt : $g'(6)$

1pt : $g''(6)$

Work for problem 4 (b) On what intervals is g $\underbrace{\text{decreasing}}_{g'<0}$? Justify your answer.

g decreasing on $[-3,0) \cup (12, 15]$

b/c $g'(x) < 0$ on $[-3,0) \cup (12,15]$

1pt – $[-3,0)$

1pt – $(12,15]$

1pt – reason

Work for problem 4 (c) On what intervals is the graph of g $\underbrace{\text{concave down}}_{g''<0 \rightarrow g' \text{ decreasing}}$? Justify your answer.

g is concave down on (6,15)

b/c g' decreasing on (6,15)

1pt - interval

1pt - reason

Work for problem 4 (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\underbrace{\Delta t = 3}_{\text{equal subintervals}}$.

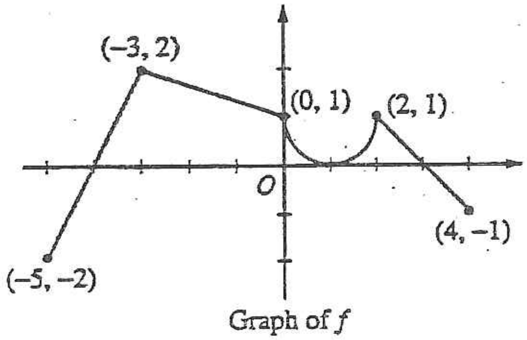
$$\int_{-3}^{15} f(t)\,dt = \frac{1}{2}(3)[-1 + 2(0) + 2(1) + 2(3) + 2(1) + 2(0) + -1]$$

← ok to stop here

$$= \frac{3}{2}[-1 + 2 + 6 + 2 + -1]$$

$$= 12$$

1pt - trap method



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

trapezoid

$g(0) = \int_{-3}^{0} f(t)\,dt$

$= \frac{1}{2}(2+1)(3)$

$\boxed{g(0) = \frac{9}{2}}$

← ok answer

$g'(x) = \frac{d}{dx}\int_{-3}^{x} f(t)\,dt$

← do not need to show this step

$g'(x) = f(x)$

$g'(0) = f(0)$

$\boxed{g'(0) = 1}$

1pt : $g(0)$

1pt : $g'(0)$

(b) Find all values of x in the open interval $(-5, 4)$ at which $\underbrace{g \text{ attains a relative maximum}}_{g' \text{ change from pos to neg.}}$. Justify your answer.

$g'(x) = f(x)$

$g'(x)$	−		+		+		−
		−4		1		3	

g has rel. max @ $x = 3$ b/c

g' changes from pos to neg @ $x = 3$.

1pt: $x = 3$

1pt: reason

NO CALCULATOR ALLOWED

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

↳ check crit #s and endpts

g has rel. min @ $x = -4$

b/c g' changes from neg to pos @ $x = -4$

1pt: identify $x = -4$ as possibility

1pt: $g(-4) = -1$

1pt: answer w/ reason (wak).

check rel. min

$$g(-4) = \int_{-3}^{-4} f(t)\,dt$$

$$= -\int_{-4}^{-3} f(t)\,dt$$

area △ ...☺

$$= -\tfrac{1}{2}(1)(2)$$

$$g(-4) = -1$$

area 2△s cancel out ...☺

check endpts

$$g(-5) = \int_{-3}^{-5} f(t)\,dt$$

$$g(-5) = 0$$

area □ area ◺ ...☺

$$g(4) = \int_{-3}^{4} f(t)\,dt = \int_{-3}^{0} f(t)\,dt + \int_{0}^{2} f(t)\,dt + \int_{2}^{4} f(t)\,dt$$

$$g(4) = \frac{9}{2} + 2(1) - \tfrac{1}{2}\pi(1)^2 + 0$$

$$g(4) = \frac{13-\pi}{2}$$

$g(-4) = -1$ → abs min value of g is −1

$g(5) = 0$

$g(4) = \frac{13-\pi}{2}$

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

g'' changes signs

$g'' = f'$

g'' + | − | − | + | −

-3 0 1 2

g has pt. of inf @ $x = -3$, $x = 1$, $x = 2$

2pts: correct values (−1 for any incorrect or missing)

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

f' changes pos to neg

↳ f' changes neg to pos.

f' + + −
1 2

1pt : rel. extremum @x=2

1pt : rel. max @x=2 w/ reason

f has rel. max @ x = 2
b/c f' changes from ~~neg~~ pos to neg @ x = 2.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

Work for problem 4(b)

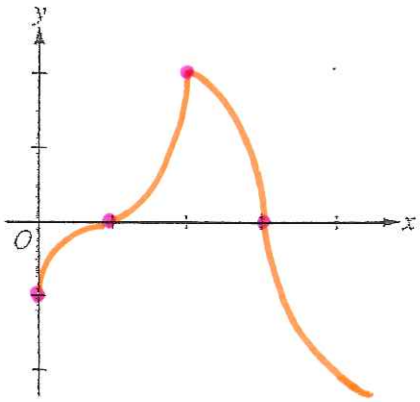
f ↗ ↗ ↘

f' + + −
1 2

f(0) = −1
f(1) = 0
f(2) = 2
f(3) = 0

f ⌢ ⌣ ⌢ ⌣

f'' − + − +
1 2 3



1pt- pts @ x = 0, 1, 2, 3 and behavior @ (2,2)

1pt- appropriate inc/dec + concavity

o not write beyond this border.

Continue problem 4 on page 11.

Work for problem 4(c)

(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

g' changes from pos to neg

g' changes from neg to pos

$g(x) = \int_1^x f(t)\,dt$

$g'(x) = f(x)$

g': − | + | −
at 1, 3

g has rel. min @ x = 1 b/c g' changes from neg to pos @ x = 1

g has rel. max @ x = 3 b/c g' changes from pos to neg @ x = 3

1pt : $g'(x) = f(x)$

1pt - crit #s

1pt - answer w/reason

Work for problem 4(d)

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

g'' changes signs

$g'(x) = f(x)$

$g''(x) = f'(x)$

f': + | + | −
at 1, 2

g has pt. of inf @ x = 2 b/c g'' changes signs @ x = 2

1pt : x = 2

1pt : answer w/reason

Do not write beyond this border.

NO CALCULATOR ALLOWED

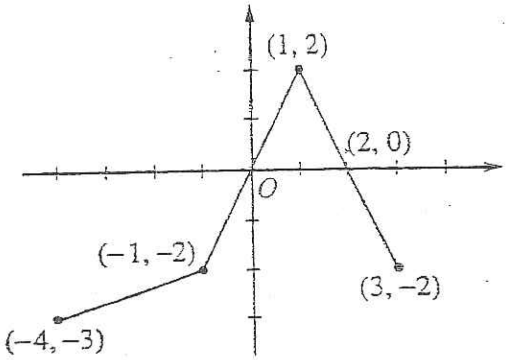
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

$g(-1) = \int_{-4}^{-1} f(t)\,dt$

trapezoid... under x-axis

$= \frac{-1}{2}(3+2)(3)$

$\boxed{g(-1) = -\frac{15}{2}}$

1pt: $g(-1)$
1pt: $g'(-1)$
1pt: $g''(-1)$

$g'(x) = \frac{d}{dx}\int_{-4}^{x} f(t)\,dt$

$g'(x) = f(x)$

$g'(-1) = f(-1)$

$\boxed{g'(-1) = -2}$

$g''(x) = f'(x)$

$g''(-1) = f'(-1)$

$\boxed{g''(-1) = \text{DNE b/c } f \text{ not diff'able @ } x = -1}$

Continue problem 4 on page 11.

Work for problem 4(b)

g" changes signs

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

$g''(x) = f'(x)$

g'' + | − (at 1)

1pt : x = 1

1pt : reason

g has inf pt @ x = 1 b/c g" changes signs @ x = 1

Work for problem 4(c)

(c) Let h be the function given by $h(x) = \int_x^3 f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

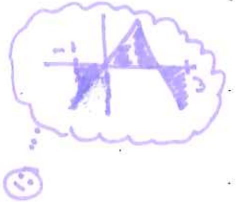
$0 = \int_x^3 f(t)\,dt$ area = 0

2pts : correct values

(−1 for missing or extra values)

x = 1

x = −1



x = 3

$\int_3^3 f(t)\,dt = 0$

$\boxed{x = 1, \; x = -1, \; x = 3}$

Work for problem 4(d)

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

$\underbrace{\text{decreasing}}_{h' < 0}$

$h(x) = \int_x^3 f(t)\,dt$

$= -\int_3^x f(t)\,dt$

$h'(x) = -\frac{d}{dx}\int_3^x f(t)\,dt$

$h'(x) = -f(x)$

h' + | − | + (at 0, 2)

1pt : interval

1pt : reason

h is decreasing on (0, 2)

b/c h' < 0 on (0, 2)

GO ON TO THE NEXT PAGE.