Topic 2: Graphs of f, f', and f''
Interpreting from Graphs & Tables of Values

There are a variety of question types here. Students may be given an equation of the function or its derivative and asked for the location of extreme values, intervals where the function is increasing or decreasing, concavity, etc. Students may be given the graph of the derivative and asked the same kinds of things. They may be asked to find the value of the integral given the graph but no equation.

This may be a particle motion problem where the velocity is given as a graph.

What students should know how to do:

- Reading information about the function from the graph of the derivative. This may be approached as a derivative techniques or antiderivative techniques.
- Find and justify extreme values (1st DT, 2nd DT, Closed interval test (aka. Candidates' test).
- Find and justify points of inflection.
- Write an equation of tangent line
- Evaluate Riemann sums from graphs only.
- FTC: Evaluate integral from area of regions on the graph.
- FTC: Realize that if $g(x)=g(a)+\int_a^t f(t)\,dt$, then $g'(x)=f(x)$
- "Family of functions": functions with a parameter;
- Functions defined by other functions.

Tables may be used to test a variety of ideas in calculus including analysis of functions, accumulation, position-velocity-acceleration, *et al.*

What students should be able to do:

- Approximate the derivative using a difference quotient.
- Use Riemann sums (left, right, midpoint) or a trapezoidal approximation to approximate the value of a definite integral using values in the table (typically with uneven subintervals). (Trapezoidal Rule, *per se*, is not required.
- Average value and the MVT may appear
- Questions about the Rolle's theorem, MVT, IVT, etc.

Do not: Use a calculator to find a regression equation and then use that to answer parts of the question. (While finding them ids perfectly good mathematics, regression equations are not one of the four things students may do with their calculator and give only an approximation of our function.)

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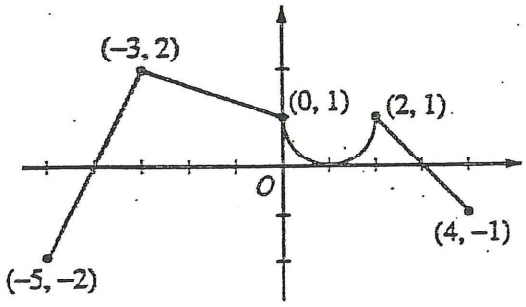


Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

$g(0) = \int_{-3}^{0} f(t)\,dt$ trapezoid

$= \frac{1}{2}(2+1)(3)$

$g(0) = \frac{9}{2}$ ok answer

$g'(x) = f(x)$

$g'(0) = f(0)$

$g'(0) = 1$

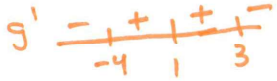
1 pt: $g(0)$

1 pt: $g'(0)$

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

g' changes from pos to neg.

$g' = f$



1 pt: $x = 3$

1 pt: reason

g has rel. max @ $x = 3$

b/c g' changes from pos to neg @ $x = 3$.

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(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

↳ check crit #s and endpts

g has rel. min @ $x = -4$
b/c g' changes from neg to pos @ $x = -4$

1 pt: identify $x=-4$ as possibility
1 pt: $g(-4) = -1$
1 pt: answer w/ reason (work)

check rel. min

$$g(-4) = \int_{-3}^{-4} f(t)\,dt = -\int_{-4}^{-3} f(t)\,dt = -\tfrac{1}{2}(1)(2) = -1$$

area △ ☺

check endpts

$$g(-5) = \int_{-3}^{-5} f(t)\,dt = 0$$

area of 2 △s cancel out ☺

$$g(4) = \int_{-3}^{4} f(t)\,dt = \int_{-3}^{0} f(t)\,dt + \int_{0}^{2} f(t)\,dt + \int_{2}^{4} f(t)\,dt$$

$$= \frac{9}{2} + 2(1) - \tfrac{1}{2}\pi(1)^2 + 0$$

$$= \frac{9}{2} + 2 - \frac{\pi}{2}$$

area of rectangle − area semicircle ☺

$\approx 4.5 + 2 - 1.5$ ☺

$g(-4) = -1$
$g(-5) = 0$
$g(4) \approx 5$

Abs min value of g is −1

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

g'' changes signs

$g'' = f'$

g'' : + | − | − | + | −
−3 0 1 2

g has inf pt @ $x = -3$, $x = 1$, $x = 2$

2 pts: correct values (−1 for incorrect or missing values)

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CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

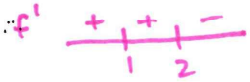
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

f' changes pos to neg

f' changes neg. to pos



1 pt: rel. extrema @ x=2

1 pt: rel max @ x=2 w/ reason

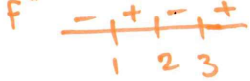
f has rel. max @ x=2

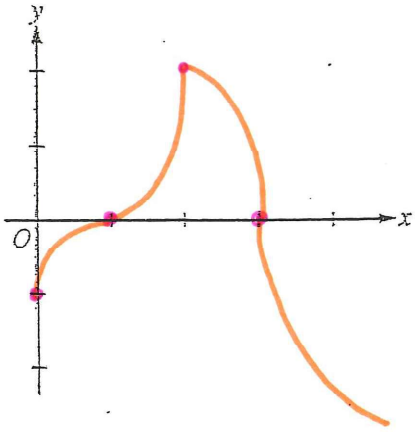
b/c f' changes from pos to neg @ x=2

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

Work for problem 4(b)

f(0) = −1
f(1) = 0
f(2) = 2
f(3) = 0





1 pt: pts @ x=0, 1, 2, 3 and behavior @ (2, 2)

1 pt: appropriate inc/dec & concavity

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Continue problem 4 on page 11.

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Work for problem 4(c)

(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

g' changes from neg to pos.

g' changes from pos to neg.

$g(x) = \int_1^x f(t)\,dt$

$g'(x) = f(x)$

g': − | + | −
1 3

1pt: $g'(x) = f(x)$

1pt: crit #s

1pt: answer w/ reason

g has rel. min @ x=1 b/c g' changes from neg to pos @ x=1

g has rel. max @ x=3 b/c g' changes from pos to neg @ x=3

Work for problem 4(d)

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

g'' changes signs

$g' = f$

$g''(x) = f'(x)$

$f' = g''$: + | + | −
1 2

1pt: x=2

1pt: answer w/ reason

g has inf pt @ x=2 b/c g'' changes signs @ x=2

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