AP Topic: Area & Volume

Given equations that define a region in the plane students are asked to find its area and the volume of the solid formed when the region is revolved around a line or used as a base of a solid with regular cross-sections. This standard application of the integral has appeared every year since 1969 on the AB exam and all but one year on the BC exam.

<u>If this appears on the calculator active section</u>: It is expected that the definite integrals will be evaluated on a calculator. Students should write the definite integral with limits on their paper and put its value after it. It is *not* required to give the antiderivative and if students give an incorrect antiderivative they will lose credit even if the final answer is (somehow) correct.

There is a calculator program available that will give the set-up and not just the answer so recently this question has been on the no calculator section. (The good news is that the integrals will be easy or they will be set-up but do not integrate questions.)

What students should know how to do:

- Find the intersection(s) of the graphs and use them as limits of integration (calculator equation solving). Write the equation followed by the solution; showing work is not required. Usually no credit until the solution is used in context.
- Find the area of the region between the graph and the *x*-axis or between two graphs.
- Find the volume when the region is revolved around a line, not necessarily and axis, by the disk/washer method. (Shell method is *never necessary* but is eligible for full credit if properly used).
- Find the volume of a solid with regular cross-sections whose base is the region between the curves. But see 2009 AB 4(b)
- Find the equation of a vertical line that divides the region in half (area or volume). This involves setting up and solving an integral equation where the limit is the variable for which the equation is solved.

CALCULUS AB
SECTION II, Part A

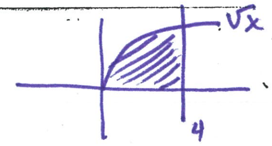
~~Time—45 minutes~~
~~Number of problems—3~~

A graphing calculator is required for some problems or parts of problems.

1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

   (a) Find the area of the region R.

Area of R $= \int_0^4 \sqrt{x}\, dx$



$= 5.333$

1 ~~2~~pt - set up
1pt - answer

(b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

$\frac{1}{2}$ Area of R $= \int_0^h \sqrt{x}\, dx$

Area of R $= 2 \int_0^h \sqrt{x}\, dx$

$5.333 = 2\left(\frac{2}{3} x^{3/2}\right)\Big|_0^h$

1 pt - equation in terms of h

$5.333 = \frac{4}{3} h^{3/2}$

1pt - answer

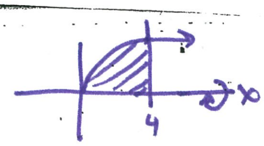
$h = 2.520$

Continue problem 1 on page 5.

(c) Find the volume of the solid generated when R is revolved about the x–axis.

$$\text{Volume} = \pi \int_0^4 (\sqrt{x})^2\, dx$$

$$= 25.133$$



DISK

1 pt - limits & constant
1 pt - integrand
1 pt - answer

(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

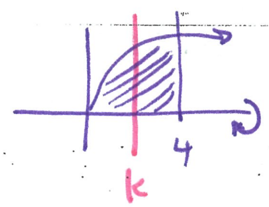
$$\frac{1}{2}\text{ Volume of } R = \pi \int_0^k (\sqrt{x})^2\, dx$$

$$\frac{1}{2}(25.133) = \pi \int_0^k (x)\, dx$$

$$\frac{25.133}{2} = \pi \left(\frac{1}{2}x^2\right)\Big|_0^k$$

$$\frac{25.133}{2} = \frac{\pi}{2}k^2$$

$$k = 2.828$$



1 ~~2~~ pts - equation in terms of k

1 pt - answer

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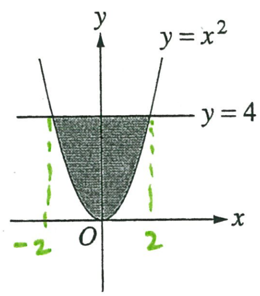
CALCULUS AB

SECTION II, Part A

~~Time—45 minutes~~

~~Number of problems—3~~

A graphing calculator is required for some problems or parts of problems.



The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

(a) Find the area of R.

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\,dx$$

$$= 10.667$$

or $\frac{32}{3}$ ... :)

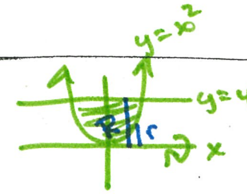
1pt - integral

1pt - answer

Do not write beyond this border.

Continue problem 1 on page 5.

(b) Find the volume of the solid generated by revolving R about the x-axis.



WASHER

$$V = \pi \int_{-2}^{2} \left(4^2 - (x^2)^2\right) dx$$

$$= 160.850$$

OR $\frac{256\pi}{5}$ ... ☺

1pt - limits + constant

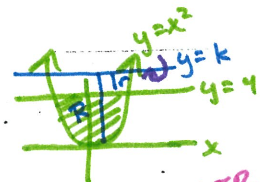
1pt - integrand

1pt - answer

(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

$$V = \pi \int_{-2}^{2} \left[(\overbrace{k - x^2}^{\text{outside radius}})^2 - (\overbrace{k-4}^{\text{inside radius}})^2 \right] dx$$

$$160.850 = \pi \int_{-2}^{2} \left[(k - x^2)^2 - (k-4)^2 \right] dx$$



WASHER

1pt - limits & constant

2pt - integrand

1pt - equation

GO ON TO THE NEXT PAGE.

CALCULUS AB
SECTION II, Part A
~~Time—45 minutes~~
~~Number of problems—3~~

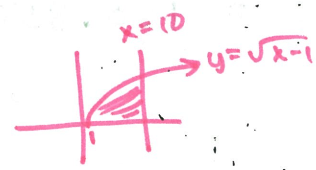
A graphing calculator is required for some problems or parts of problems.

1. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

   (a) Find the area of R.

Area of $R = \int_{1}^{10} \sqrt{x-1}\, dx$

$= 18$

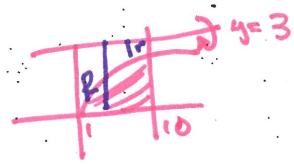


1pt - limits
1pt - integrand
1pt - answer

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

Volume $= \pi \int_{1}^{10} \left((3-0)^2 - (3-\sqrt{x-1})^2 \right) dx$

$= 212.058$



WASHER

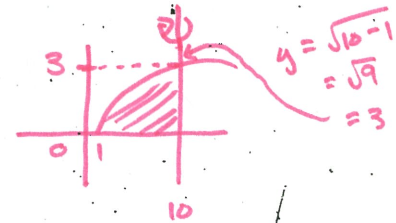
1pt - limits + constant

1pt - integrand

1pt - answer

Continue problem 1 on page 5.

(c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.



DISK
(in terms of y's)

$$V = \pi \int_0^3 \left(10 - (y^2+1)\right)^2 \, dy$$

$= 407.150$

$y = \sqrt{x-1}$

$y^2 = x - 1$

$y^2 + 1 = x$

1pt - limits & constant

1pt - integrand

1pt - answer

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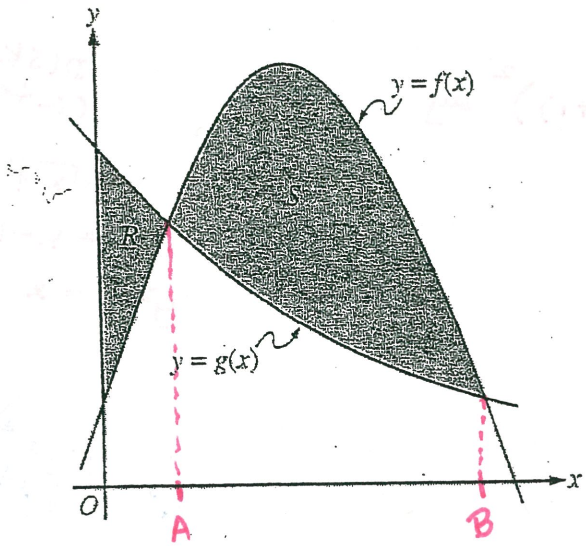
CALCULUS AB

SECTION II, Part A

~~Time—45 minutes~~

~~Number of problems—3~~

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

$A = 0.178$

Area of $R = \int_0^A (g(x) - f(x))\,dx$

$= 0.065$

1pt - limits
1pt - integrand
1pt - answer

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Continue problem 1 on page 5.

(b) Find the area of S.

$A = 0.178$

$B = 1$

Area of $S = \int_A^B (f(x) - g(x))\, dx$

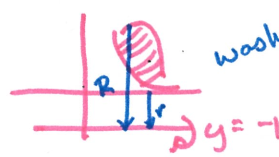
$= 0.410$

1pt- limits

1pt - integ

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

$V = \pi \int_A^B \left[(f(x) - -1)^2 - (g(x) - -1)^2 \right] dx$



washer

$= 4.559$

2 pts- integrand,

1 pt - limits, constant, + answer

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