AP Calculus – Final Review Sheet

When you see the words ….	**This is what you think of doing**
1. Find the zeros	Set function = 0, factor or use quadratic equation if quadratic, graph to find zeros on calculator
2. Find equation of the line tangent to $f(x)$ on $[a,b]$	Take derivative - $f'(a)=m$ and use $y-y_1=m(x-x_1)$
3. Find equation of the line normal to $f(x)$ on $[a,b]$	Same as above but $m=\dfrac{-1}{f'(a)}$
4. Show that $f(x)$ is even	Show that $f(-x)=f(x)$ - symmetric to *y*-axis
5. Show that $f(x)$ is odd	Show that $f(-x)=-f(x)$ - symmetric to origin
6. Find the interval where $f(x)$ is increasing	Find $f'(x)$, set both numerator and denominator to zero to find critical points, make sign chart of $f'(x)$ and determine where it is positive.
7. Find interval where the slope of $f(x)$ is increasing	Find the derivative of $f'(x)=f''(x)$, set both numerator and denominator to zero to find critical points, make sign chart of $f''(x)$ and determine where it is positive.
8. Find the minimum value of a function	Make a sign chart of $f'(x)$, find all relative minimums and plug those values back into $f(x)$ and choose the smallest.
9. Find the minimum slope of a function	Make a sign chart of the derivative of $f'(x)=f''(x)$, find all relative minimums and plug those values back into $f'(x)$ and choose the smallest.
10. Find critical values	Express $f'(x)$ as a fraction and set both numerator and denominator equal to zero.
11. Find inflection points	Express $f''(x)$ as a fraction and set both numerator and denominator equal to zero. Make sign chart of $f''(x)$ to find where $f''(x)$ changes sign. (+ to – or – to +)
12. Show that $\lim_{x\to a} f(x)$ exists	Show that $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)$
13. Show that $f(x)$ is continuous	Show that 1) $\lim_{x\to a} f(x)$ exists ($\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)$) 2) $f(a)$ exists 3) $\lim_{x\to a} f(x)=f(a)$
14. Find vertical asymptotes of $f(x)$	Do all factor/cancel of $f(x)$ and set denominator = 0
15. Find horizontal asymptotes of $f(x)$	Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
16. Find the average rate of change of $f(x)$ on $[a,b]$	Find $\dfrac{f(b)-f(a)}{b-a}$
17. Find instantaneous rate of change of $f(x)$ at a	Find $f'(a)$

18. Find the average value of $f(x)$ on $[a,b]$	Find $\dfrac{\int_a^b f(x)\,dx}{b-a}$
19. Find the absolute maximum of $f(x)$ on $[a,b]$	Make a sign chart of $f'(x)$, find all relative maximums and plug those values back into $f(x)$ as well as finding $f(a)$ and $f(b)$ and choose the largest.
20. Show that a piecewise function is differentiable at the point *a* where the function rule splits	First, be sure that the function is continuous at $x=a$. Take the derivative of each piece and show that $\lim\limits_{x\to a^-} f'(x) = \lim\limits_{x\to a+} f'(x)$
21. Given $s(t)$ (position function), find $v(t)$	Find $v(t)=s'(t)$
22. Given $v(t)$, find how far a particle travels on $[a,b]$	Find $\int_a^b \lvert v(t)\rvert dt$
23. Find the average velocity of a particle on $[a,b]$	Find $\dfrac{\int_a^b v(t)\,dt}{b-a} = \dfrac{s(b)-s(a)}{b-a}$
24. Given $v(t)$, determine if a particle is speeding up at $t=k$	Find $v(k)$ and $a(k)$. Multiply their signs. If both positive, the particle is speeding up, if different signs, then the particle is slowing down.
25. Given $v(t)$ and $s(0)$, find $s(t)$	$s(t)=\int v(t)\,dt + C$ Plug in $t=0$ to find C
26. Show that Rolle's Theorem holds on $[a,b]$	Show that *f* is continuous and differentiable on the interval. If $f(a)=f(b)$, then find some *c* in $[a,b]$ such that $f'(c)=0$.
27. Show that Mean Value Theorem holds on $[a,b]$	Show that *f* is continuous and differentiable on the interval. Then find some *c* such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.
28. Find domain of $f(x)$	Assume domain is $(-\infty,\infty)$. Restrictable domains: denominators $\neq 0$, square roots of only non negative numbers, log or ln of only positive numbers.
29. Find range of $f(x)$ on $[a,b]$	Use max/min techniques to rind relative max/mins. Then examine $f(a), f(b)$
30. Find range of $f(x)$ on $(-\infty,\infty)$	Use max/min techniques to rind relative max/mins. Then examine $\lim\limits_{x\to\pm\infty} f(x)$.
31. Find $f'(x)$ by definition	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ or $f'(x)=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$
32. Find derivative of inverse to $f(x)$ at $x=a$	Interchange *x* with *y*. Solve for $\dfrac{dy}{dx}$ implicitly (in terms of *y*). Plug your *x* value into the inverse relation and solve for *y*. Finally, plug that *y* into your $\dfrac{dy}{dx}$.

33. y is increasing proportionally to y	$\frac{dy}{dt} = ky$ translating to $y = Ce^{kt}$
34. Find the line $x = c$ that divides the area under $f(x)$ on $[a,b]$ to two equal areas	$\int_a^c f(x)dx = \int_c^b f(x)dx$
35. $\frac{d}{dx}\int_a^x f(t)\,dt =$	2nd FTC: Answer is $f(x)$
36. $\frac{d}{dx}\int_a^u f(t)\,dt$	2nd FTC: Answer is $f(u)\frac{du}{dx}$
37. The rate of change of population is …	$\frac{dP}{dt} = \ldots$
38. The line $y = mx + b$ is tangent to $f(x)$ at (x_1, y_1)	Two relationships are true. The two functions share the same slope ($m = f'(x)$) and share the same y value at x_1.
39. Find area using left Riemann sums	$A = base[x_0 + x_1 + x_2 + \ldots + x_{n-1}]$
40. Find area using right Riemann sums	$A = base[x_1 + x_2 + x_3 + \ldots + x_n]$
41. Find area using midpoint rectangles	Typically done with a table of values. Be sure to use only values that are given. If you are given 6 sets of points, you can only do 3 midpoint rectangles.
42. Find area using trapezoids	$A = \frac{base}{2}[x_0 + 2x_1 + 2x_2 + \ldots + 2x_{n-1} + x_n]$ This formula only works when the base is the same. If not, you have to do individual trapezoids.
43. Solve the differential equation …	Separate the variables – *x* on one side, *y* on the other. The *dx* and *dy* must all be upstairs.
44. Meaning of $\int_a^x f(t)\,dt$	The accumulation function – accumulated area under the function $f(x)$ starting at some constant *a* and ending at *x*.
45. Given a base, cross sections perpendicular to the *x*-axis are squares	The area between the curves typically is the base of your square. So the volume is $\int_a^b (base^2)dx$
46. Find where the tangent line to $f(x)$ is horizontal	Write $f'(x)$ as a fraction. Set the numerator equal to zero.
47. Find where the tangent line to $f(x)$ is vertical	Write $f'(x)$ as a fraction. Set the denominator equal to zero.
48. Find the minimum acceleration given $v(t)$	First find the acceleration $a(t) = v'(t)$. Then minimize the acceleration by examining $a'(t)$.
49. Approximate the value of $f(0.1)$ by using the tangent line to *f* at $x = 0$	Find the equation of the tangent line to *f* using $y - y_1 = m(x - x_1)$ where $m = f'(0)$ and the point is $(0, f(0))$. Then plug in 0.1 into this line being sure to use an approximate $(\approx)$ sign.

50. Given the value of $F(a)$ and the fact that the anti-derivative of *f* is *F*, find $F(b)$ 1	Usually, this problem contains an antiderivative you cannot take. Utilize the fact that if $F(x)$ is the antiderivative of *f*, then $\int_a^b F(x)dx = F(b) - F(a)$. So solve for $F(b)$ using the calculator to find the definite integral.
51. Find the derivative of $f(g(x))$	$f'(g(x)) \cdot g'(x)$
52. Given $\int_a^b f(x)\,dx$, find $\int_a^b [f(x)+k]\,dx$	$\int_a^b [f(x)+k]\,dx = \int_a^b f(x)dx + \int_a^b k dx$
53. Given a picture of $f'(x)$, find where $f(x)$ is increasing	Make a sign chart of $f'(x)$ and determine where $f'(x)$ is positive.
54. Given $v(t)$ and $s(0)$, find the greatest distance from the origin of a particle on $[a,b]$	Generate a sign chart of $v(t)$ to find turning points. Then integrate $v(t)$ using $s(0)$ to find the constant to find $s(t)$. Finally, find *s*(all turning points) which will give you the distance from your starting point. Adjust for the origin.
55. Given a water tank with *g* gallons initially being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$, find a) the amount of water in the tank at *m* minutes	$g + \int_t^{t_2} (F(t) - E(t))dt$
56. b) the rate the water amount is changing at *m*	$\frac{d}{dt}\int_t^m (F(t) - E(t))dt = F(m) - E(m)$
57. c) the time when the water is at a minimum	$F(m) - E(m)$=0, testing the endpoints as well.
58. Given a chart of *x* and $f(x)$ on selected values between *a* and *b*, estimate $f'(c)$ where *c* is between *a* and b.	Straddle *c*, using a value *k* greater than *c* and a value *h* less than *c*. so $f'(c) \approx \frac{f(k) - f(h)}{k - h}$
59. Given $\frac{dy}{dx}$, draw a slope field	Use the given points and plug them into $\frac{dy}{dx}$, drawing little lines with the indicated slopes at the points.
60. Find the area between curves $f(x), g(x)$ on $[a,b]$	$A = \int_a^b [f(x) - g(x)]dx$, assuming that the *f* curve is above the *g* curve.
61. Find the volume if the area between $f(x), g(x)$ is rotated about the *x*-axis	$A = \int_a^b [(f(x))^2 - (g(x))^2]dx$ assuming that the *f* curve is above the *g* curve.

BC Problems

62. Find $\lim_{x\to\infty}\frac{f(x)}{g(x)}$ if $\lim_{x\to\infty} f(x)=\lim_{x\to\infty} g(x)=0$	Use L'Hopital's Rule.
63. Find $\int_0^{\infty} f(x)\ dx$	$\lim_{h\to\infty}\int_0^h f(x)dx$
64. $\frac{dP}{dt}=\frac{k}{M}P(M-P)$ or $\frac{dP}{dt}=kP\left(1-\frac{P}{M}\right)$	Signals logistic growth. $\lim_{t\to\infty}\frac{dP}{dt}=0\Rightarrow M=P$
65. Find $\int\frac{dx}{x^2+ax+b}$ where x^2+ax+b factors	Factor denominator and use Heaviside partial fraction technique.
66. The position vector of a particle moving in the plane is $r(t)=\langle x(t),y(t)\rangle$ a) Find the velocity.	$v(t)=\langle x'(t),y'(t)\rangle$
67. b) Find the acceleration.	$a(t)=\langle x''(t),y''(t)\rangle$
68. c) Find the speed.	$\lVert v(t)\rVert=\sqrt{[x'(t)]^2+[y'(t)]^2}$
69. a) Given the velocity vector $v(t)=\langle x(t),y(t)\rangle$ and position at time 0, find the position vector.	$s(t)=\int x(t)dt+\int y(t)dt+C$ Use $s(0)$ to find *C,* remembering it is a vector.
70. b) When does the particle stop?	$v(t)=0\to x(t)=0$ AND $y(t)=0$
71. c) Find the slope of the tangent line to the vector at t_1.	This is the acceleration vector at t_1.
72. Find the area inside the polar curve $r=f(\theta)$.	$A=\frac{1}{2}\int_{\theta_1}^{\theta_2}[f(\theta)]^2 d\theta$
73. Find the slope of the tangent line to the polar curve $r=f(\theta)$.	$x=r\cos\theta, y=r\sin\theta\Rightarrow\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$
74. Use Euler's method to approximate $f(1.2)$ given $\frac{dy}{dx}$, $(x_0,y_0)=(1,1)$, and $\Delta x=0.1$	$dy=\frac{dy}{dx}(\Delta x)$, $y_{\text{new}}=y_{\text{old}}+dy$
75. Is the Euler's approximation an underestimate or an overestimate?	Look at sign of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the interval. This gives you increasing.decreasing/concavity. Draw picture to ascertain

	answer.
76. Find $\int x^n e^{ax} dx$ where a, n are integers	Integration by parts, $\int u\,dv = uv - \int v\,du + C$
77. Write a series for $x^n \cos x$ where n is an integer	$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$ Multiply each term by x^n
78. Write a series for $\ln(1+x)$ centered at $x=0$.	Find Maclaurin polynomial: $P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$
79. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if…..	$p > 1$
80. If $f(x) = 2 + 6x + 18x^2 + 54x^3 + ...$, find $f\left(-\frac{1}{2}\right)$	Plug in and factor. This will be a geometric series: $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$
81. Find the interval of convergence of a series.	Use a test (usually the ratio) to find the interval and then test convergence at the endpoints.
82. Let S_4 be the sum of the first 4 terms of an alternating series for $f(x)$. Approximate $\|f(x) - S_4\|$	This is the error for the 4th term of an alternating series which is simply the 5th term. It will be positive since you are looking for an absolute value.
83. Suppose $f^{(n)}(x) = \frac{(n+1)\ \ n!}{2^n}$. Write the first four terms and the general term of a series for $f(x)$ centered at $x = c$	You are being given a formula for the derivative of $f(x)$. $f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$
84. Given a Taylor series, find the Lagrange form of the remainder for the n^{th} term where n is an integer at $x = c$.	You need to determine the largest value of the 5th derivative of f at some value of z. Usually you are told this. Then: $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$
85. $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$	$f(x) = e^x$
86. $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + ... + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + ...$	$f(x) = \sin x$
87. $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... + \frac{(-1)^n x^{2n}}{(2n)!} + ...$	$f(x) = \cos x$
88. Find $\int (\sin x)^m (\cos x)^n dx$ where m and n are integers	If m is odd and positive, save a sine and convert everything else to cosine. The sine will be the du. If n is odd and positive, save a cosine and convert everything else to sine. The cosine will be the du. Otherwise use the fact that:

	$\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$
89. Given $x = f(t), y = g(t)$, find $\frac{dy}{dx}$	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$
90. Given $x = f(t), y = g(t)$, find $\frac{d^2y}{dx^2}$	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$
91. Given $f(x)$, find arc length on $[a,b]$	$L = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx$
92. $x = f(t), y = g(t)$, find arc length on $[t_1, t_2]$	$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
93. Find horizontal tangents to a polar curve $r = f(\theta)$	$x = r\cos\theta, y = r\sin\theta$ Find where $r\sin\theta = 0$ where $r\cos\theta \neq 0$
94. Find vertical tangents to a polar curve $r = f(\theta)$	$x = r\cos\theta, y = r\sin\theta$ Find where $r\cos\theta = 0$ where $r\sin\theta \neq 0$
95. Find the volume when the area between $y = f(x), x = 0, y = 0$ is rotated about the y-axis.	Shell method: $V = 2\pi \int_0^b$ radius $\cdot$ height dx where b is the root.
96. Given a set of points, estimate the volume under the curve using Simpson's rule on $[a,b]$.	$A \approx \frac{b-a}{3n}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + ... + 4y_{n-1} + y_n\right]$
97. Find the dot product: $\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle$	$\langle u_1, u_2\rangle \cdot \langle v_1, v_2\rangle = u_1v_1 + u_2v_2$
98. Multiply two vectors:	You get a scalar.