CALCULUS BC

SECTION I, Part A

Time—55 minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. If $y = \sin(3x)$, then $\frac{dy}{dx} =$

(A) $-3\cos(3x)$ (B) $-\cos(3x)$ (C) $-\frac{1}{3}\cos(3x)$ (D) $\cos(3x)$ (E) $3\cos(3x)$

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2. $\lim_{x \to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x}$ is

(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

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3. $\int (3x + 1)^5 dx =$

(A) $\frac{(3x + 1)^6}{18} + C$

(B) $\frac{(3x + 1)^6}{6} + C$

(C) $\frac{(3x + 1)^6}{2} + C$

(D) $\frac{\left(\frac{3x^2}{2} + x\right)^6}{2} + C$

(E) $\left(\frac{3x^2}{2} + x\right)^5 + C$

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4. For $0 \le t \le 13$, an object travels along an elliptical path given by the parametric equations $x = 3 \cos t$ and $y = 4 \sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

(A) $-\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $-\frac{4 \tan 13}{3}$ (D) $-\frac{4}{3 \tan 13}$ (E) $-\frac{3}{4 \tan 13}$

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5. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5 ?

(A) 3 (B) 5 (C) 6 (D) 10 (E) 12

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6. What are all values of p for which $\int_1^{\infty} \frac{1}{x^{2p}}\,dx$ converges?

(A) $p < -1$

(B) $p > 0$

(C) $p > \frac{1}{2}$

(D) $p > 1$

(E) There are no values of p for which this integral converges.

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7. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

(A) -1 only (B) 0 only (C) 2 only (D) -1 and 2 only (E) -1, 0, and 2

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8. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

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9. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

10. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1 (B) 2 (C) 4 (D) 6 (E) The series diverges.

Section I
Part A

11. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

(A) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$

(B) $x^2 + x^3 + x^4 + x^5 + \cdots$

(C) $x^2 + 2x^3 + 3x^4 + 4x^5 + \cdots$

(D) $x^2 + x^4 + x^6 + x^8 + \cdots$

(E) $x^2 - x^4 + x^6 - x^8 + \cdots$

12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

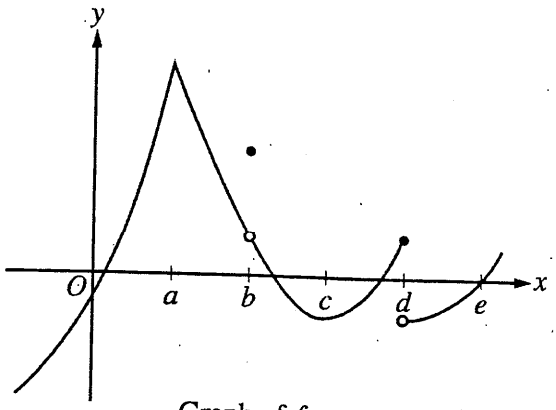
(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

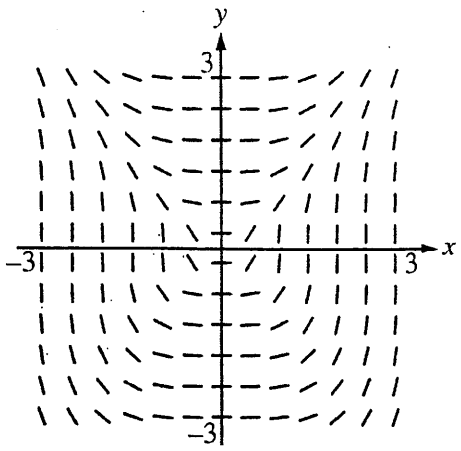
(E) $\frac{dV}{dt} = k\sqrt{V}$



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e



14. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = \frac{x^2}{y^2}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\frac{dy}{dx} = \frac{x^2}{y}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

15. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1 + 9x^4}\,dx$. If the curve contains the point $(1, 6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$

(B) $y = 5 + x^3$

(C) $y = 6 + x^3$

(D) $y = 6 - x^3$

(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

17. A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3, 8)$?

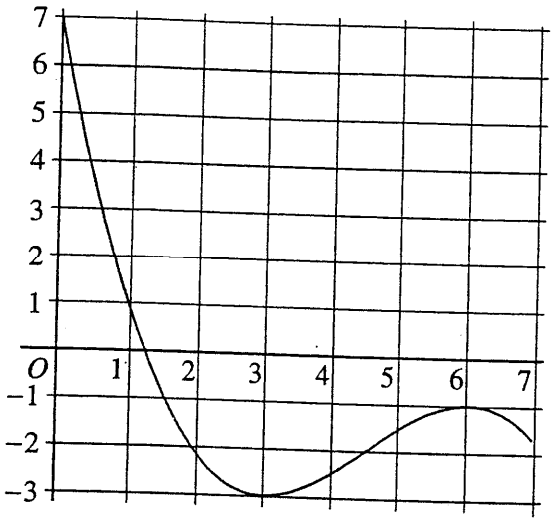
(A) $x = -3$

(B) $x = 2$

(C) $y = 8$

(D) $y = -\frac{27}{10}(x + 3) + 8$

(E) $y = 12(x + 3) + 8$



Graph of f

18. The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0 (B) −1 (C) −2 (D) −3 (E) −6

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point (1, 2) ?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

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20. A function f has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots + \frac{x^{n+3}}{(n+1)!} + \cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x \sin x + 3x^2$

(B) $-\cos\left(x^2\right) + 1$

(C) $-x^2 \cos x + x^2$

(D) $x^2 e^x - x^3 - x^2$

(E) $e^{x^2} - x^2 - 1$

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21. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t \to \infty} M(t)$?

(A) 50 (B) 200 (C) 500 (D) 1000 (E) 2000

22. What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges?

(A) $p > 0$ (B) $p \geq 1$ (C) $p > 1$ (D) $p \geq 2$ (E) $p > 2$

23. $\int x \sin(6x)\,dx =$

(A) $-x\cos(6x) + \sin(6x) + C$

(B) $-\frac{x}{6}\cos(6x) + \frac{1}{36}\sin(6x) + C$

(C) $-\frac{x}{6}\cos(6x) + \frac{1}{6}\sin(6x) + C$

(D) $\frac{x}{6}\cos(6x) + \frac{1}{36}\sin(6x) + C$

(E) $6x\cos(6x) - \sin(6x) + C$

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24. Which of the following series diverge?

I. $\sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$

II. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$

III. $\sum_{n=1}^{\infty}\left(\frac{e^n}{e^n + 1}\right)$

(A) III only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

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x	2	5	10	14
$f(x)$	12	28	34	30

25. The function f is continuous on the closed interval $[2, 14]$ and has values as shown in the table above. Using the subintervals $[2, 5]$, $[5, 10]$, and $[10, 14]$, what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum?

(A) 296 (B) 312 (C) 343 (D) 374 (E) 390

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26. $\displaystyle\int \frac{2x}{(x+2)(x+1)}\,dx =$

(A) $\ln|x+2| + \ln|x+1| + C$

(B) $\ln|x+2| + \ln|x+1| - 3x + C$

(C) $-4\ln|x+2| + 2\ln|x+1| + C$

(D) $4\ln|x+2| - 2\ln|x+1| + C$

(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

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27. $\frac{d}{dx}\left(\int_0^{x^3} \ln\left(t^2+1\right) dt\right) =$

(A) $\frac{2x^3}{x^6+1}$ (B) $\frac{3x^2}{x^6+1}$ (C) $\ln\left(x^6+1\right)$ (D) $2x^3\ln\left(x^6+1\right)$ (E) $3x^2\ln\left(x^6+1\right)$

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28. What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x = 0$?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) 6
