2003 - BC

(1) $y = \sin(3x)$

$\frac{dy}{dx} = \cos(3x) \cdot 3$

$= 3\cos(3x)$

$\boxed{E}$

(2) $\lim_{x\to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x} = \frac{1-1-0}{0-0} = \frac{0}{0}$ L'Hopital

$\lim_{x\to 0} \frac{e^x + \sin x - 2}{2x - 2} = \frac{1+0-2}{0-2} = \frac{-1}{-2}$

$\frac{1}{2}$

$\boxed{C}$

(3) $\int (3x+1)^5 \, dx$ $\quad u = 3x+1$, $\frac{du}{dx} = 3$, $\frac{du}{3} = dx$

$= \int u^5 \cdot \frac{du}{3}$

$= \frac{1}{3}\int u^5 \, du$

$= \frac{1}{3}\left(\frac{1}{6}u^6\right) + C$

$= \frac{1}{18}(3x+1)^6 + C$ $\boxed{A}$

(4) $\frac{dx}{dt} = -3\sin t$ $\quad \left.\frac{dx}{dt}\right|_{t=13} = -3\sin 13$

$\frac{dy}{dt} = 4\cos t$ $\quad \left.\frac{dy}{dt}\right|_{t=13} = 4\cos 13$

$\left.\frac{dy/dt}{dx/dt}\right|_{t=13} = \frac{4\cos 13}{-3\sin 13} = -\frac{4}{3}\cot 13$

OR $\frac{-4}{3\tan 13}$

$\boxed{D}$

(5) Eulers Method $\rightarrow y - y_1 = m(x - x_1)$
$y = y + m(\Delta x)$

	dy/dx	$\Delta x\left(\frac{dy}{dx}\right)$	$y + \Delta x\left(\frac{dy}{dx}\right)$
(1, 2)	3	.5(3) = 1.5	2 + 1.5 = 3.5
(1.5, 3.5)	5	.5(5) = 2.5	3.5 + 2.5 = 6
(2, 6)			

$\boxed{C}$

(6) $\int_1^\infty \frac{1}{x^{2p}} dx$ converges

when $\sum_{n=1}^\infty \frac{1}{x^{2p}}$ converges

by p-test, if $2p > 1$

$p > \frac{1}{2}$

$\boxed{C}$

(7) particle rest when velocity = 0

$\frac{dx}{dt} = 3t^2 - 6t$
$3t(t-2) = 0 \Rightarrow t = 0, t = 2$

$\frac{dy}{dt} = 6t^2 - 6t - 12$
$6(t^2 - t - 2)$
$6(t-2)(t+1) = 0 \Rightarrow t = 2, t = -1$

rest when both $\frac{dy}{dt}$ + $\frac{dx}{dt} = 0$

@ $t = 2$ $\boxed{C}$

(8) $\int x^2 \cos(x^3) \, dx$ $\quad u = x^3$, $\frac{du}{dx} = 3x^2$, $\frac{du}{3x^2} = dx$

$\int x^2 \cdot \cos u \cdot \frac{du}{3x^2}$

$\frac{1}{3}\int \cos u \, du$

$\frac{1}{3}\sin u + C$

$\frac{1}{3}\sin(x^3) + C$ $\boxed{B}$

(9) $f(x) = \ln(x + 4 + e^{-3x})$

$f'(x) = \frac{1}{x+4+e^{-3x}} \cdot (1 + e^{-3x} \cdot -3)$

$f'(0) = \frac{1}{4+e^0}(1 + e^0(-3))$

$= \frac{1}{5}(1-3)$

$= -\frac{2}{5}$

$\boxed{A}$

(10) $\sum_{n=1}^\infty \frac{2^{n+1}}{3^n} = \sum_{n=1}^\infty \left(\frac{2}{3}\right)^n \cdot 2$ = geometric

sum $= \frac{a_1}{1-r}$

$= \frac{2\left(\frac{2}{3}\right)^1}{1 - 2/3}$

$= \frac{4/3}{1/3}$

$= 4$

$\boxed{C}$

(11) $\frac{1}{1-x} = \sum_{n=0}^\infty x^n$

$x^2\left(\frac{1}{1-x^2}\right) = x^2\left(\sum_{n=0}^\infty (x^2)^n\right)$

$= x^2\left(\sum_{n=0}^\infty x^{2n}\right)$

$x^2(1 + x^2 + (x^2)^2 + (x^2)^3 + \dots)$

$x^2 + x^4 + x^6 + x^8 + \dots$

$\boxed{D}$

(12) rate of change V $\rightarrow \frac{dV}{dt}$

directly proportion $\rightarrow k(\ )$

square root Volume $\rightarrow \sqrt{V}$

$\frac{dV}{dt} = k\sqrt{V}$

$\boxed{E}$

(13) f cont $\rightarrow$ no jumps, no holes, no asymptotes $\quad$ a, c

f not diff'able $\rightarrow$ sharp turn, vertical tangent line $\quad$ a

$\boxed{A}$

(14) $\frac{dy}{dx} = 0$ @ $x = 0$

$\frac{dy}{dx}$ DNE @ $y = 0$

when $x < 0, y < 0$, $\frac{dy}{dx} < 0$ (QUAD III), D or E

when $x < 0, y > 0$, $\frac{dy}{dx} < 0$ (QUAD II), E

$\boxed{E}$

15. $L = \int_1^4 \sqrt{1+9x^4}\,dx$

$= \int_1^4 \sqrt{1+(3x^2)^2}\,dx$

$\frac{dy}{dx} = 3x^2$

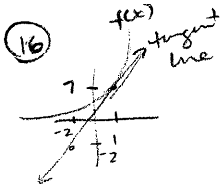
$\int dy = \int 3x^2\,dx$

$y = x^3 + C$

$6 = 1^3 + C$

$5 = C$

$\boxed{y = x^3+5}$ $\boxed{B}$

16. 

$f'(1) = \frac{7 - -2}{1 - -2}$

$= \frac{9}{3}$

$= 3$

$\boxed{C}$

17. $\frac{dx}{dt} = 2t - 4$ $\quad y = t^3$

$\frac{dy}{dt} = 3t^2$ $\quad 8 = t^3$

$2 = t$

$\left.\frac{dy}{dx}\right|_{t=2} = \frac{3(2)^2}{2(2)-4} = \frac{12}{0} = \text{DNE}$

$x = -3$

$\boxed{A}$

18. $g(x) = \int_0^{2x} f(t)\,dt$

$g'(x) = \frac{d}{dx}\int_0^{2x} f(t)\,dt$

$g'(x) = f(2x)\cdot 2$

$g'(3) = f(6)\cdot 2$

$= -1(2)$

$= -2$

$\boxed{C}$

19. $\frac{dy}{dx} = 2x+3$

$\int dy = \int (2x+3)\,dx$

$y = x^2+3x+C$

$2 = 1^2 + 3(1) + C$

$2 = 4 + C$

$-2 = C$

$y = x^2+3x-2$

$\boxed{D}$

20. $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots$

$x^2\left(\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$

$x^2(e^x - 1 - x)$ $\quad e^x = 1 + x + \frac{x^2}{2!} + \cdots$

$e^x - 1 - x = \frac{x^2}{2!} + \cdots$

$x^2e^x - x^2 - x^3$

$\boxed{D}$

21. $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$

$= 0.6M\left(\frac{1}{200}\right)(200-M)$

$\lim_{t\to\infty} M(t) = 200$

$\boxed{B}$

22. $\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ *LCT*

$a_n = \frac{n}{n^p+1}$ compares to $b_n = \frac{n}{n^p}$

$\lim_{n\to\infty} \frac{\frac{n}{n^p+1}}{\frac{n}{n^p}} = \lim_{n\to\infty} \frac{n^p}{n^p+1} = 1$ constant, so

$\sum \frac{n}{n^p} = \sum \frac{1}{n^{p-1}}$

converges when $p-1>1$ (p-test)

$p > 2$

$\boxed{E}$

23. $\int x\sin(6x)\,dx$

by parts,

$u = x \quad dv = \sin(6x)$

$du = dx \quad v = -\frac{1}{6}\cos(6x)$

$x\left(-\frac{1}{6}\cos(6x)\right) - \int -\frac{1}{6}\cos 6x\,dx$

$-\frac{1}{6}x\cos 6x + \frac{1}{6}\int \cos 6x\,dx$

$-\frac{1}{6}x\cos 6x + \frac{1}{36}\sin 6x + C$

$\boxed{B}$

24. I. geo series

✗ $\frac{\sin 2}{\pi} < 1$

so converges

II. p-test with $p = \frac{1}{3}$

✓ $\frac{1}{3} < 1$ diverges

III. $e^n + 1 > e^n$ *DCT*

$\frac{1}{e^n} > \frac{1}{e^n+1}$ Since $\sum 1$ diverges

$\frac{e^n}{e^n} > \frac{e^n}{e^n+1}$ then

$1 > \frac{e^n}{e^n+1}$ $\sum \frac{e^n}{e^n+1}$ also diverges

$\boxed{D}$

25. $\int_2^{14} f(x)\,dx = \Delta x(30) + \Delta x(34) + \Delta x(28)$

$= 4(30) + 5(34) + 3(28)$

$= 120 + 170 + 84$

$= 290 + 84$

$= 374$

$\boxed{D}$

26. $\int \frac{2x}{(x+2)(x+1)}\,dx$

$= \int \frac{4}{x+2}\,dx + \int \frac{-2}{x+1}\,dx$

$= 4\ln|x+2| - 2\ln|x+1| + C$

$\boxed{D}$

partial fractions

$\frac{2x}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$2x = Ax + A + Bx + 2B$

$2 = A + B \quad A + 2B = 0$

$0 = -A - 2B \quad A + -4 = 0$

$2 = -B \quad A = 4$

$-2 = B$

27. $\frac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)\,dt\right)$

$\ln((x^3)^2+1)\cdot 3x^2$

$3x^2\cdot\ln(x^6+1)$

$\boxed{E}$

28. $\frac{1}{(1+x)^2} = ?$ $\quad \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$

$\frac{d}{dx}\left(\frac{1}{1+x}\right) = \frac{d}{dx}(1 - x + x^2 - x^3 + \cdots)$

$-1(1+x)^{-2} = -1 + 2x - 3x^2 + \cdots$

$(1+x)^{-2} = 1 - 2x + 3x^2 - \cdots$

$\boxed{D}$

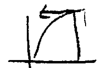
29) trapezoid goes over, right rectangles under

$\boxed{A}$

right rect. under yes!

trap over, yes!

(B)



right rect not under

No!

(C)

trap not over

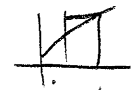
No!

(D)

right rect not under

No!

(E)



right rect not under

No!

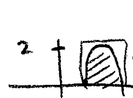
30) $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$

$\frac{1}{2}\int_2^4 f(t)\,dt = 1$

$\int_2^4 f(t)\,dt = 2$

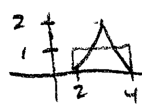
need area from $x = 2$ ~~to~~ $x = 4$ to be 2.

(A)



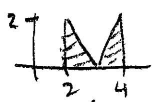
→ area is a little less than $2(2) = 4$

(B)



→ area is close to $2(1) = 2$

(C)



Area $= \frac{1}{2}(1)(2) + \frac{1}{2}(1)(2)$

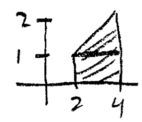
$= 1 + 1$

$= 2$

Exactly 2

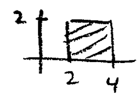
$\boxed{C}$

(D)



area $= \frac{1}{2}(2)(1) + 2(1) = 3$

(E)



area $= 2(2)$

$= 4$