2008

Section I Answer Key and Percent Answering Correctly
Calculus BC

	Item No.	Correct Answer	Percent Correct by Grade					Total Percent Correct
			5	4	3	2	1	
	1	B	97	95	93	89	81	93
*	2	A	96	92	87	80	67	89
	3	C	88	77	67	55	36	73
	4	D	95	86	74	61	40	80
	5	C	94	85	76	66	40	80
*	6	A	81	61	52	43	35	63
	7	D	90	73	58	43	23	69
*	8	B	89	71	56	39	26	68
*	9	D	96	85	74	58	33	79
*	10	B	90	80	72	59	39	76
	11	A	75	48	34	27	21	52
	12	D	64	35	25	21	16	42
*	13	B	75	55	40	29	23	55
*	14	E	63	39	32	27	22	45
*	15	A	85	63	45	31	17	61
	16	D	62	38	27	21	16	42
*	17	B	79	60	45	37	28	59
*	18	A	59	33	23	16	13	38
	19	A	61	41	30	23	18	43
	20	C	49	23	16	12	9	30
*	21	A	58	35	23	16	10	38
	22	E	31	11	7	6	5	18
	23	E	80	57	42	28	15	56
	24	A	63	37	25	19	13	41
*	25	B	57	27	19	13	12	35
	26	D	56	35	25	19	13	38
*	27	A	83	66	52	41	28	64
	28	D	26	5	3	2	2	13

* AB Questions

2008 BC ...

① $a(t) = v'(t)$

$= \langle 2t, 5\rangle$

$a(3) = \langle 6, 5\rangle$ $\boxed{B}$

② $\int x e^{x^2}\,dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\int x e^{u} \cdot \frac{du}{2x}$

$= \frac{1}{2}\int e^{u}\,du$

$= \frac{1}{2}e^{u} + C$

$= \frac{1}{2}e^{x^2} + C$ $\boxed{A}$

③ $\lim_{x\to 0} \frac{\sin x \cos x}{x}$

$= \left(\lim_{x\to 0} \frac{\sin x}{x}\right)\left(\lim_{x\to 0} \cos x\right)$

$= 1 \cdot \cos 0$

$= 1$

$\boxed{C}$

④ $\lim_{n\to\infty} \frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^{n}}$

$\lim_{n\to\infty} \frac{e \cdot n!}{(n+1)!}$

$\lim_{n\to\infty} \frac{e \cdot \cancel{n!}}{(n+1)\cancel{n!}} = \lim_{n\to\infty} \frac{e}{n+1} < 1$

$\boxed{D}$

⑤ $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= \int_0^{\pi} \sqrt{(\cos(t^3) \cdot 3t^2)^2 + (e^{5t} \cdot 5)^2}\,dt$

$= \int_0^{\pi} \sqrt{9t^4\cos^2(t^3) + 25e^{10t}}\,dt$

$\boxed{C}$

⑥ I. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ ✓

× II. $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$

$= 4$

$f(2) = 1$

$\lim_{x\to 2} f(x) \neq f(2)$ ∴ f not cont @ $x = 2$

× III. f not cont @ $x = 2$,

∴ f not diff'able @ $x = 2$

I only $\boxed{A}$

⑦

	Δx	$\frac{dy}{dx}$	$\Delta x \frac{dy}{dx}$	$y + \Delta x \frac{dy}{dx}$
$(1,-3)$	$.5$	-1	$-.5$	$-3 + -.5 = -3.5$
$(1.5,-3.5)$	$.5$	$-.5$	$-.25$	$-3.5 + -.25 = -3.75$
$(2,-3.75)$				

$f(2) = -3.75$ $\boxed{D}$

⑧ left sum

$\int_2^{13} f(x)\,dx = 1(6) + 2(-2) + 3(-1) + 5(3)$

$= 6 - 4 - 3 + 15$

$\boxed{B}$ $= 14$

⑨ greatest → max → g' change from pos to neg.

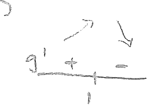
$g'(x) = 0$

$g'(x) = \frac{d}{dx}\int_{-2}^{x} f(t)\,dt$

$g'(x) = f(x)$

$0 = f(x)$

@ $x = 1$



$\boxed{D}$ g(1) max – no need to check endpts b/c g(−3) and g(2) are lower than g(1)

⑩ $x^2 + xy + y^2 = 7$ @ $(2,1)$

$2x + y(1) + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

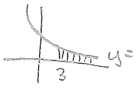
$2(2) + 1 + 2\frac{dy}{dx} + 2\frac{dy}{dx} = 0$

$5 + 4\frac{dy}{dx} = 0$

$4\frac{dy}{dx} = -5$

$\boxed{B}$ $\frac{dy}{dx} = \frac{-5}{4}$

⑪



$y = e^{-2x}$ Area $= \int_3^{\infty} e^{-2x}\,dx$

$= \lim_{a\to\infty} \int_3^{a} e^{-2x}\,dx$

$= \lim_{a\to\infty} -\frac{1}{2}e^{-2x}\Big|_3^a = \lim_{a\to\infty} -\frac{1}{2e^{2x}}\Big|_3^a$

$= \lim_{a\to\infty}\left(-\frac{1}{2e^{2a}} - -\frac{1}{2e^6}\right) = \frac{1}{2e^6}$ $\boxed{A}$

⑫ (A) $\lim_{n\to\infty} \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n}$

$= \lim_{n\to\infty} x\left(\frac{n}{n+1}\right)$

$= x$

so, converges on $|x| < 1$

(B) $\lim_{n\to\infty} \frac{x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{x^n}$

$= \lim_{n\to\infty} x\left(\frac{n^2}{(n+1)^2}\right)$

$= x$

so, converges on $|x| < 1$

(C) $\lim_{n\to\infty} \frac{x^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{x^n}$

$= \lim_{n\to\infty} x\left(\frac{\sqrt{n}}{\sqrt{n+1}}\right)$

$= x$

so, converges on $|x| < 1$

$\boxed{D}$ (D) $\lim_{n\to\infty} \frac{e^{n+1} \cdot x^{n+1}}{(n+1)!} \cdot \frac{n!}{e^{n} \cdot x^{n}}$

$= \lim_{n\to\infty} e \cdot x\left(\frac{\cancel{n!}}{(n+1)\cancel{n!}}\right)$

$= 0$, so converges ∀ x

(E) $\lim_{n\to\infty} \frac{(n+1)!\,x^{n+1}}{e^{n+1}} \cdot \frac{e^{n}}{n!\,x^{n}}$

$\lim_{n\to\infty} \frac{x}{e}\left(\frac{(n+1)\cancel{n!}}{\cancel{n!}}\right) = \infty$, diverges

⑬ $\int_1^{e} \frac{x^2+1}{x}\,dx$

$= \int_1^{e}\left(\frac{x^2}{x} + \frac{1}{x}\right)dx$

$= \int_1^{e}\left(x + \frac{1}{x}\right)dx$

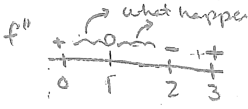
$= \left(\frac{1}{2}x^2 + \ln|x|\right)\Big|_1^e$

$= \frac{1}{2}e^2 + \ln e - \left(\frac{1}{2}(1)^2 + \ln 1\right)$

$= \frac{1}{2}e^2 + 1 - \frac{1}{2} + 0$

$\boxed{B}$ $= \frac{1}{2}e^2 + \frac{1}{2}$

⑭



what happens btn (0,1) and btn (1,2)?

(A) f inc → f' > 0
no info about f'

(B) f dec → f' < 0
no info about f'

(C) f rel max @ x = 1, so, f'(1) = 0 and f''(1) < 0
not true, f''(1) = 0 and no info about f'(1)

(D) inf pt @ x = 1, f'' changes signs ... maybe, but signs on (0,1) and (1,2) unknown

(E) f changes concavity on (0,2)
yes ... b/c f''(x) > 0 @ x = 0
f''(x) < 0 @ x = 2

$\boxed{E}$

⑮ $f(x) = (\ln x)^2$

$f'(x) = 2(\ln x) \cdot \frac{1}{x}$

$= \frac{2}{x}\ln x$

$f''(x) = \ln x(-2x^{-2}) + \frac{2}{x}\left(\frac{1}{x}\right)$

$= -\frac{2\ln x}{x^2} + \frac{2}{x^2}$

$f''(\sqrt{e}) = -\frac{2\ln\sqrt{e}}{(\sqrt{e})^2} + \frac{2}{(\sqrt{e})^2}$

$= \frac{-2\left(\frac{1}{2}\right)}{e} + \frac{2}{e}$

$= -\frac{1}{e} + \frac{2}{e} = \frac{1}{e}$ $\boxed{A}$

⑯ $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$

geometric
converges $|r|<1$

$\left|\frac{2}{x^2+1}\right|<1$

$2<|x^2+1|$

$|x^2+1|>2$

$x^2+1>2$ $\quad$ $x^2+1<-2$

$x^2>1$ $\quad$ $\cancel{x^2<-1}$

$|x|>1$

$x>1,\ x<-1$ $\quad\boxed{D}$

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⑰ $F'(x)=h'(x^2-3)\cdot 2x$

$F'(2)=h'(2^2-3)\cdot 2(2)$

$=h'(1)\cdot 4$

$\boxed{B}$ $\quad =4h'(1)$

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⑱ $x+y=k$

$y=-x+k$

$m=-1$

$y=x^2+3x+1$

$\frac{dy}{dx}=2x+3$

$2x+3=-1$

$2x=-4$

$x=-2$

$y=(-2)^2+3(-2)+1$

$=4-6+1$

$y=-1$

$x+y=k$

$-2+-1=k$

$-3=k$ $\quad\boxed{A}$

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⑲ $\int\frac{7x}{(2x-3)(x+2)}dx$

$\frac{7x}{(2x-3)(x+2)}=\frac{A}{2x-3}+\frac{B}{x+2}$

$7x=Ax+2A+2Bx-3B$

$0=2A-3B$ $\quad$ $7=A+2B$

$-14=-2A-4B$ $\quad$ $7=A+2(2)$

$-14=-7B$ $\quad$ $7=A+4$

$2=B$ $\quad$ $3=A$

$\int\frac{3}{2x-3}dx+\int\frac{2}{x+2}dx$

$3\cdot\frac{1}{2}\ln|2x-3|+2\ln|x+2|+C$

$\boxed{A}$

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⑳ $1+\ln 2+\frac{(\ln 2)^2}{2!}+\ldots\frac{(\ln 2)^n}{n!}+\ldots$

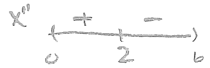
$e^x=1+x+\frac{x^2}{2!}+\ldots\frac{x^n}{n!}+\ldots$

$e^{\ln 2}=1+\ln 2+\frac{(\ln 2)^2}{2!}+\ldots\frac{(\ln 2)^n}{n!}$

$e^{\ln 2}=2$ $\quad\boxed{C}$

㉑ $v(t)$ inc $\Rightarrow$ $a(t)$ positive
or $x''(t)$ positive

$x''(t)$ pos when $x(t)$ concave up



on $(0,2)$ $\quad\boxed{A}$

㉒ $\int_0^1 f(x)g'(x)dx = uv-\int v\,du$

$u=f(x)$ $\quad$ $\int dv=\int g'(x)dx$

$du=f'(x)dx$ $\quad$ $v=g(x)$

$=f(x)g(x)\Big|_0^1-\int_0^1 g(x)\cdot f'(x)dx$

$=f(1)g(1)-f(0)g(0)-5$

$=4(3)-2(-4)-5$

$=12+8-5$

$=15$

$\boxed{E}$

㉓ $f(x)=x\sin(2x)$

$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots$

$\sin(2x)=2x-\frac{(2x)^3}{3!}+\frac{(2x)^5}{5!}+\ldots$

$=2x-\frac{8x^3}{3!}+\frac{32x^5}{5!}+\ldots$

$x\sin(2x)=2x^2-\frac{8x^4}{3!}+\frac{32x^6}{5!}+\ldots$

$\boxed{E}$

㉔ $\frac{dP}{dt}=kP(M-P)$

↖ carrying capacity
$M=200$

$\boxed{A}$

(A) $\frac{dP}{dt}=.001P(200-P)$

(B) $\frac{dP}{dt}=.001P(100-P)$

(C) $\frac{dP}{dt}=.001P(200P-1)$

(D) $\frac{dP}{dt}=.001P(100P-1)$

(E) $\frac{dP}{dt}=.001P(100P+1)$

㉕ $f(x)=\begin{cases}cx+d & x\le 2\\ x^2-cx & x>2\end{cases}$ $\quad$ $f'(x)=\begin{cases}c & x\le 2\\ 2x-c & x>2\end{cases}$

$\lim_{x\to 2^-}(cx+d)=\lim_{x\to 2^+}(x^2-cx)$ $\quad$ $\lim_{x\to 2^-}c=\lim_{x\to 2^+}(2x-c)$

$2c+d=4-2c$ $\quad$ $c=4-c$

$4+d=4-4$ $\quad$ $2c=4$

$4+d=0$ $\quad$ $c=2$

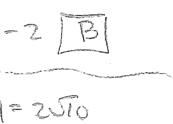
$d=-4$

$c+d=2+(-4)=-2$ $\quad\boxed{B}$

㉖ $\sin^2\theta=0$

$\sin\theta=0$

$\theta=0,\pi,2\pi,\ldots$



$A=\frac{1}{2}\int_0^{2\pi}(\sin^2\theta)^2\,d\theta$

$=\frac{1}{2}\int_0^{2\pi}(\sin^4\theta)\,d\theta$

or $2\left(\frac{1}{2}\int_0^{\pi}(\sin^4\theta)\,d\theta\right)$

$\boxed{D}$ $\quad\int_0^{\pi}(\sin^4\theta)\,d\theta$

㉗ $\frac{dy}{dx}=y^2-1$

Depends only on y,
so, not B, C, or D.
when $y=\pm 1$, $\frac{dy}{dx}=0$,
So, $\boxed{A}$

㉘ $y=x^2-x$ $\quad$ speed $=|v(t)|=2\sqrt{10}$

$\frac{dy}{dt}=?$

$|v(t)|=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$

$\frac{dy}{dt}=2x\frac{dx}{dt}-\frac{dx}{dt}$

$=(2x-1)\frac{dx}{dt}$

$2\sqrt{10}=\sqrt{\left(\frac{dx}{dt}\right)^2+\left((2x-1)\frac{dx}{dt}\right)^2}$

$2\sqrt{10}=\sqrt{\left(\frac{dx}{dt}\right)^2}\sqrt{1+(2x-1)^2}$

$2\sqrt{10}=\frac{dx}{dt}\sqrt{1+(2x-1)^2}$

$\frac{2\sqrt{10}}{\sqrt{1+(2\cdot 2-1)^2}}=\frac{dx}{dt}\Big|_{x=2}$

$\frac{2\sqrt{10}}{\sqrt{10}}=\frac{dx}{dt}\Big|_{x=2}$

$\frac{dx}{dt}\Big|_{x=2}=2$

$\frac{dy}{dt}\Big|_{x=2}=(2(2)-1)(2)$

$=3(2)$

$=6$

$\boxed{D}$