**CALCULUS BC**
**SECTION I, Part A**
**Time—55 minutes**
**Number of questions—28**

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

**Directions:** Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

**In this exam:**

(1)   Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

(2)   The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$ ).

1. If $f(x) = \dfrac{x^2 + 3x + 2}{x + 3}$, then $f'(x) =$

    (A) $2x + 3$

    (B) $\dfrac{-x^2 - 6x - 7}{(x + 3)^2}$

    (C) $\dfrac{x^2 + 6x + 7}{(x + 3)^2}$

    (D) $\dfrac{x^2 + 12x + 11}{(x + 3)^2}$

    (E) $\dfrac{3x^2 + 12x + 11}{(x + 3)^2}$

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2. $\displaystyle\int 5x\left(\sqrt{x} - x^2\right) dx =$

    (A) $\dfrac{15\sqrt{x}}{2} - 15x^2 + C$

    (B) $5x - \dfrac{5x^4}{4} + C$

    (C) $2x^{5/2} - \dfrac{5x^4}{4} + C$

    (D) $\dfrac{25x^{5/2}}{2} - \dfrac{5x^4}{4} + C$

    (E) $\dfrac{5x^{7/2}}{3} - \dfrac{5x^6}{6} + C$

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3. What is the value of $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$ ?

(A) $-\dfrac{15}{8}$     (B) $-\dfrac{9}{8}$     (C) $-\dfrac{3}{8}$     (D) $\dfrac{9}{8}$     (E) $\dfrac{15}{8}$

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4. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$ ?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\dfrac{7}{3}(x - 1)$

(C) $y + 3 = \dfrac{1}{3}(x - 1)$

(D) $y + 3 = \dfrac{7}{3}(x - 1)$

(E) $y + 3 = \dfrac{11}{3}(x - 1)$

5. If $y = \dfrac{1}{2}x^{4/5} - \dfrac{3}{x^5}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2}{5x^{1/5}} + \dfrac{15}{x^6}$

(B) $\dfrac{2}{5x^{1/5}} + \dfrac{15}{x^4}$

(C) $\dfrac{2}{5x^{1/5}} - \dfrac{3}{5x^4}$

(D) $\dfrac{2x^{1/5}}{5} + \dfrac{15}{x^6}$

(E) $\dfrac{2x^{1/5}}{5} - \dfrac{3}{5x^4}$

Graph of $f$

6. The graph of the function $f$ consists of two line segments, as shown in the figure above. The value of $\int_0^3 |f(x)|\, dx$ is

(A) $-\dfrac{3}{2}$　　(B) $\dfrac{1}{2}$　　(C) $\dfrac{3}{2}$　　(D) $\dfrac{5}{2}$　　(E) nonexistent

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7. A population $y$ changes at a rate modeled by the differential equation $\dfrac{dy}{dt} = 0.2y(1000 - y)$, where $t$ is measured in years. What are all values of $y$ for which the population is increasing at a decreasing rate?

(A) 500 only

(B) $0 < y < 500$ only

(C) $500 < y < 1000$ only

(D) $0 < y < 1000$

(E) $y > 1000$

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8. Which of the following gives the length of the path described by the parametric equations $x(t) = 2 + 3t$ and $y(t) = 1 + t^2$ from $t = 0$ to $t = 1$?

(A) $\displaystyle\int_0^1 \sqrt{1 + \dfrac{4t^2}{9}}\ dt$

(B) $\displaystyle\int_0^1 \sqrt{1 + 4t^2}\ dt$

(C) $\displaystyle\int_0^1 \sqrt{3 + 3t + t^2}\ dt$

(D) $\displaystyle\int_0^1 \sqrt{9 + 4t^2}\ dt$

(E) $\displaystyle\int_0^1 \sqrt{(2 + 3t)^2 + \left(1 + t^2\right)^2}\ dt$

9. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$?

(A) 0        (B) 1        (C) 2.75        (D) 3        (E) 6

10. If $\displaystyle\int_0^k \dfrac{x}{x^2 + 4}\, dx = \dfrac{1}{2}\ln 4$, where $k > 0$, then $k =$

(A) 0        (B) $\sqrt{2}$        (C) 2        (D) $\sqrt{12}$        (E) $\dfrac{1}{2}\tan\left(\ln\sqrt{2}\right)$

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11. The third-degree Taylor polynomial for a function $f$ about $x = 4$ is $\dfrac{(x-4)^3}{512} - \dfrac{(x-4)^2}{64} + \dfrac{(x-4)}{4} + 2$. What is the value of $f'''(4)$?

(A) $-\dfrac{1}{64}$     (B) $-\dfrac{1}{32}$     (C) $\dfrac{1}{512}$     (D) $\dfrac{3}{256}$     (E) $\dfrac{81}{256}$

12. For which of the following does $\lim\limits_{x\to\infty} f(x) = 0$?

    I. $f(x) = \dfrac{\ln x}{x^{99}}$

    II. $f(x) = \dfrac{e^x}{\ln x}$

    III. $f(x) = \dfrac{x^{99}}{e^x}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

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Graph of $f'$

13. The graph of $f'$, the derivative of $f$, is shown in the figure above. If $f(0) = 20$, which of the following could be the value of $f(5)$ ?

(A) 15     (B) 20     (C) 25     (D) 35     (E) 40

14. If $a$ and $b$ are positive constants, then $\displaystyle\lim_{x\to\infty} \frac{\ln(bx+1)}{\ln(ax^2+3)} =$

(A) 0      (B) $\dfrac{1}{2}$      (C) $\dfrac{1}{2}ab$      (D) 2      (E) $\infty$

15. What are all values of $x$ for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\left(x+\frac{3}{2}\right)^n$ converges?

(A) $-\dfrac{5}{2} < x < -\dfrac{1}{2}$

(B) $-\dfrac{5}{2} < x \le -\dfrac{1}{2}$

(C) $-\dfrac{5}{2} \le x < -\dfrac{1}{2}$

(D) $-\dfrac{1}{2} < x < \dfrac{1}{2}$

(E) $x \le -\dfrac{1}{2}$

16. For $0 < P < 100,$ which of the following is an antiderivative of $\dfrac{1}{100P - P^2}$ ?

(A) $\dfrac{1}{100}\ln(P) - \dfrac{1}{100}\ln(100 - P)$

(B) $\dfrac{1}{100}\ln(P) + \dfrac{1}{100}\ln(100 - P)$

(C) $100\ln(P) - 100\ln(100 - P)$

(D) $\ln\left(100P - P^2\right)$

(E) $\dfrac{1}{50P^2 - \dfrac{P^3}{3}}$

17. If $\displaystyle\lim_{h \to 0} \dfrac{\arcsin(a + h) - \arcsin(a)}{h} = 2,$ which of the following could be the value of $a$ ?

(A) $\dfrac{\sqrt{2}}{2}$      (B) $\dfrac{\sqrt{3}}{2}$      (C) $\sqrt{3}$      (D) $\dfrac{1}{2}$      (E) 2

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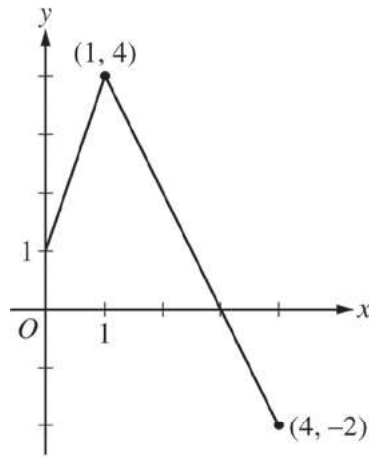
Graph of $f$

18. The graph of the function $f$, consisting of two line segments, is shown in the figure above. Let $g$ be the function given by $g(x) = 2x + 1$, and let $h$ be the function given by $h(x) = f(g(x))$. What is the value of $h'(1)$ ?

(A) $-4$      (B) $-2$      (C) 4      (D) 6      (E) nonexistent

19. Which of the following is the Maclaurin series for $\dfrac{1}{(1-x)^2}$ ?
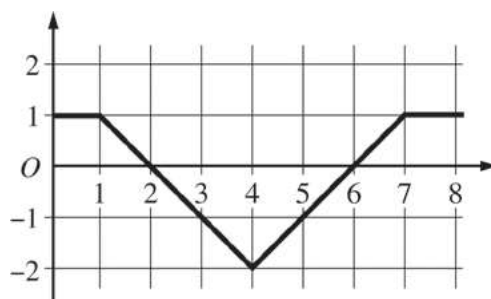
(A) $1 - x + x^2 - x^3 + \cdots$

(B) $1 - 2x + 3x^2 - 4x^3 + \cdots$

(C) $1 + 2x + 3x^2 + 4x^3 + \cdots$

(D) $1 + x^2 + x^4 + x^6 + \cdots$

(E) $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \dfrac{x^4}{4} + \cdots$

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Graph of $f$

20. The graph of the function $f$ in the figure above consists of four line segments. Let $g$ be the function defined by

$g(x) = \int_0^x f(t)\, dt$. Which of the following is an equation of the line tangent to the graph of $g$ at $x = 5$ ?

(A) $y + 1 = x - 5$

(B) $y - 2 = x - 5$

(C) $y - 2 = -1(x - 5)$

(D) $y + 2 = x - 5$

(E) $y + 2 = -1(x - 5)$

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21. At time $t \geq 0$, a cube has volume $V(t)$ and edges of length $x(t)$. If the volume of the cube decreases at a rate proportional to its surface area, which of the following differential equations could describe the rate at which the volume of the cube decreases?

(A) $\dfrac{dV}{dt} = -1.2x^2$

(B) $\dfrac{dV}{dt} = -1.2x^3$

(C) $\dfrac{dV}{dt} = -1.2x^2t$

(D) $\dfrac{dV}{dt} = -1.2t^2$

(E) $\dfrac{dV}{dt} = -1.2V^2$

22. Which of the following is true about the curve $x^2 - xy + y^2 = 3$ at the point $(2, 1)$ ?

(A) $\dfrac{dy}{dx}$ exists at $(2, 1)$, but there is no tangent line at that point.

(B) $\dfrac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is horizontal.

(C) $\dfrac{dy}{dx}$ exists at $(2, 1)$, and the tangent line at that point is neither horizontal nor vertical.

(D) $\dfrac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is vertical.

(E) $\dfrac{dy}{dx}$ does not exist at $(2, 1)$, and the tangent line at that point is horizontal.

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23. What is the coefficient of $x^6$ in the Taylor series for $\dfrac{e^{3x^2}}{2}$ about $x = 0$ ?

(A) $\dfrac{1}{1440}$     (B) $\dfrac{81}{160}$     (C) $\dfrac{9}{4}$     (D) $\dfrac{9}{2}$     (E) $\dfrac{27}{2}$

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24. The function $g$ is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of $g$ on the closed interval $[-2, 1]$ ?

(A) $-7$      (B) $-\dfrac{3}{4}$      (C) $0$      (D) $2$      (E) $6$

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25. Which of the following is the solution to the differential equation $\dfrac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$ ?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

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26. Which of the following series converge?

 I. $\displaystyle\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$  II. $\displaystyle\sum_{n=1}^{\infty} e^{-n}$  III. $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

27. If $\displaystyle\int_1^x f(t)\,dt = \frac{20x}{\sqrt{4x^2+21}} - 4$, then $\displaystyle\int_1^\infty f(t)\,dt$ is

(A) 6  (B) 1  (C) −3  (D) −4  (E) divergent

28. If $x = t^2 - 1$ and $y = \ln t$, what is $\dfrac{d^2 y}{dx^2}$ in terms of $t$ ?

(A) $-\dfrac{1}{2t^4}$      (B) $\dfrac{1}{2t^4}$      (C) $-\dfrac{1}{t^3}$      (D) $-\dfrac{1}{2t^2}$      (E) $\dfrac{1}{2t^2}$

**END OF PART A OF SECTION I**

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.**

**DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.**

## CALCULUS BC
### SECTION I, Part B
### Time—50 minutes
### Number of questions—17

### A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
### THIS PART OF THE EXAM.

**Directions:** Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

**BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.**

**YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.**

**In this exam:**

(1)   The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2)   Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

(3)   The inverse of a trigonometric function $f$ may be indicated using the inverse function notation $f^{-1}$ or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$ ).

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76. Let $f$ be a function whose derivative is given by $f'(x) = \ln\left(x^4 + 5x^3 + x^2 - 7x + 28\right)$. On the open interval $(-4, 1)$, at which of the following values of $x$ does $f$ attain a relative maximum?

(A) $-3.623$ only

(B) $-0.871$ only

(C) $-3.623$ and $-3.284$

(D) $-3.459$ and $0.581$ only

(E) $-3.459$, $-0.871$, and $0.581$

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| $a$ | $\lim\limits_{x \to a^-} f(x)$ | $\lim\limits_{x \to a^+} f(x)$ | $f(a)$ |
|-----|------|------|------|
| −1 | 4 | 6 | 4 |
| 0 | −3 | −3 | 5 |
| 1 | 2 | 2 | 2 |

77. The function $f$ has the properties indicated in the table above. Which of the following must be true?

(A) $f$ is continuous at $x = -1$

(B) $f$ is continuous at $x = 0$

(C) $f$ is continuous at $x = 1$

(D) $f$ is differentiable at $x = 0$

(E) $f$ is differentiable at $x = 1$.

78. What is the area of the region in the first quadrant enclosed by the graphs of $y = \sin(2x)$ and $y = x$

(A) 0.208     (B) 0.210     (C) 0.266     (D) 0.660     (E) 0.835

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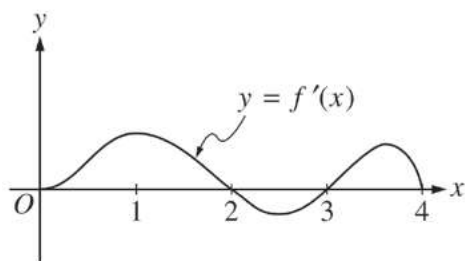
79. The figure above shows the graph of $f'$, the derivative of the function $f$. If $f(0) = 0$, which of the following could be the graph of $f$?

(A)



(B)



(C)



(D)



(E)

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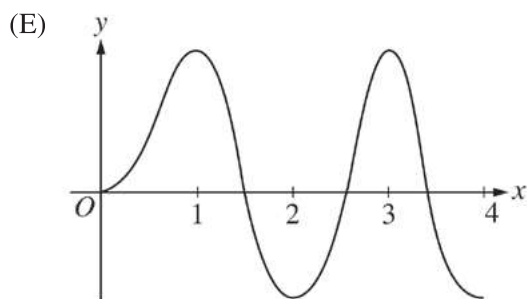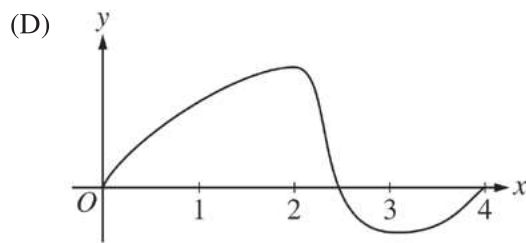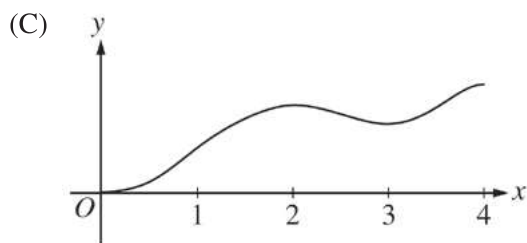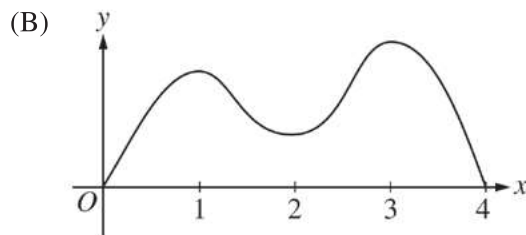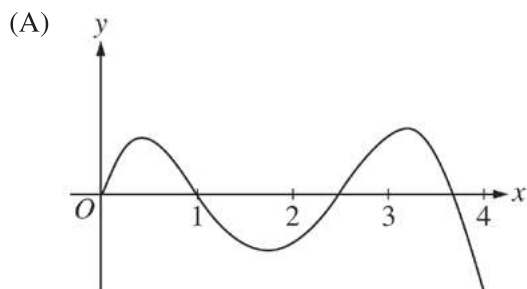
80. The volume of a certain cone for which the sum of its radius, $r$, and height is constant is given by $V = \frac{1}{3}\pi r^2 (10 - r)$. The rate of change of the radius of the cone with respect to time is 6. In terms of $r$, what is the rate of change of the volume of the cone with respect to time?

(A) $-24\pi r$      (B) $6\pi r$      (C) $\frac{20}{3}\pi r - \pi r^2$      (D) $16\pi r - \frac{4}{3}\pi r^2$      (E) $40\pi r - 6\pi r^2$

81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit $(°\text{F})$. If the initial temperature of the tea, at time $t = 0$ minutes, is $200°\text{F}$ and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) $175°\text{F}$      (B) $130°\text{F}$      (C) $95°\text{F}$      (D) $70°\text{F}$      (E) $45°\text{F}$

82. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 4 | 10 | 18 | 28 | 40 | 54 |

83. The table above gives selected values for a continuous function $f$. If $f$ is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

(A) 50        (B) 62        (C) 77        (D) 100        (E) 154

84. Let $f$ be a function with derivative given by $f'(x) = x^3 - 5x^2 + e^x$. On which of the following intervals is the graph of $f$ concave down?

(A) $(-\infty, 0.117)$ only

(B) $(-\infty, 1.144)$

(C) $(0.116, 2.062)$

(D) $(0.673, 2.863)$

(E) $(2.863, \infty)$

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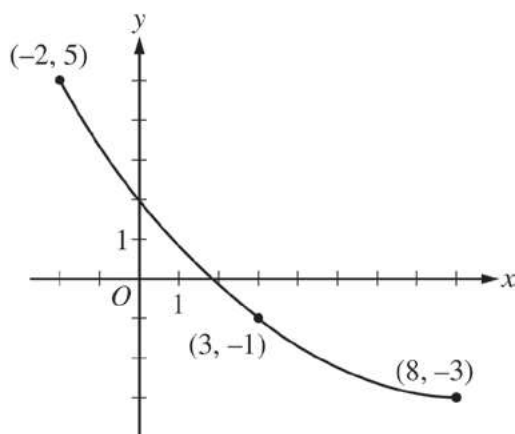
Graph of $f$

85. A portion of the graph of a differentiable function $f$ is shown above. If the value $c = 3$ satisfies the conclusion of the Mean Value Theorem applied to $f$ on the open interval $-2 < x < 8$, what is the slope of the line tangent to the graph of $f$ at $x = 3$ ?

(A) $-\dfrac{7}{5}$        (B) $-\dfrac{5}{4}$        (C) $-\dfrac{4}{5}$        (D) $-\dfrac{5}{7}$        (E) $-\dfrac{1}{5}$

86. If $f'(x) > 0$ for all $x$ and $f''(x) < 0$ for all $x$, which of the following could be a table of values for $f$?

(A)

| $x$ | $f(x)$ |
|-----|--------|
| −1  | 4      |
| 0   | 3      |
| 1   | 1      |

(B)

| $x$ | $f(x)$ |
|-----|--------|
| −1  | 4      |
| 0   | 4      |
| 1   | 4      |

(C)

| $x$ | $f(x)$ |
|-----|--------|
| −1  | 4      |
| 0   | 5      |
| 1   | 6      |

(D)

| $x$ | $f(x)$ |
|-----|--------|
| −1  | 4      |
| 0   | 5      |
| 1   | 7      |

(E)

| $x$ | $f(x)$ |
|-----|--------|
| −1  | 4      |
| 0   | 6      |
| 1   | 7      |

87. The position of a particle moving in the $xy$-plane is given by the parametric functions $x(t)$ and $y(t)$ for which $x'(t) = t \sin t$ and $y'(t) = 5e^{-3t} + 2.$ What is the slope of the line tangent to the path of the particle at the point at which $t = 2$ ?

(A) 0.904  (B) 1.107  (C) 1.819  (D) 2.012  (E) 3.660
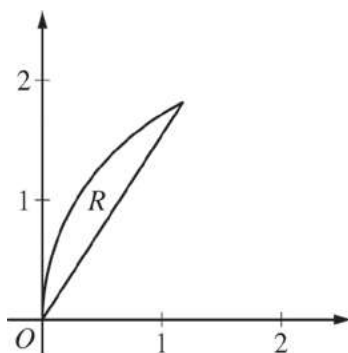
88. Let $R$ be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos\theta$ and below by the line $\theta = 1$, as shown in the figure above. What is the area of $R$ ?

(A) 0.317     (B) 0.465     (C) 0.929     (D) 2.618     (E) 5.819

89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the $y$-axis?

(A) 3.464     (B) 4.500     (C) 7.854     (D) 10.883     (E) 14.137

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90. Which of the following statements are true about the series $\displaystyle\sum_{n=2}^{\infty} a_n$, where $a_n = \dfrac{(-1)^n}{\sqrt{n} + (-1)^n}$ ?

    I. The series is alternating.

    II. $\left|a_{n+1}\right| \leq \left|a_n\right|$ for all $n \geq 2$

    III. $\displaystyle\lim_{n \to \infty} a_n = 0$

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

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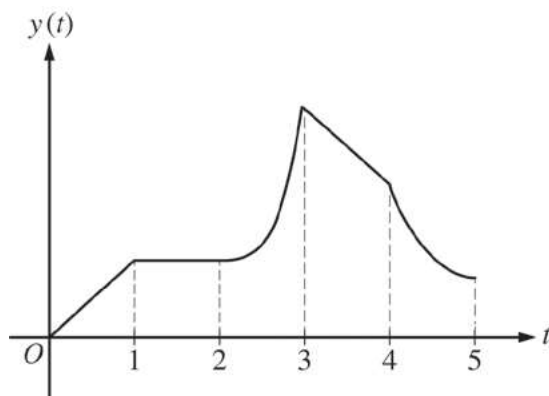
91. A particle moves along the *y*-axis. The graph of the particle's position $y(t)$ at time $t$ is shown above for $0 \le t \le 5$. For what values of $t$ is the velocity of the particle negative and the acceleration positive?

(A) $0 < t < 1$　　(B) $1 < t < 2$　　(C) $2 < t < 3$　　(D) $3 < t < 4$　　(E) $4 < t < 5$

92. If $f$ is a function such that $f'(x) = -f(x)$, then $\int x f(x)\, dx =$

(A) $f(x)(x + 1) + C$

(B) $-f(x)(x + 1) + C$

(C) $\dfrac{x^2}{2} f(x) + C$

(D) $-\dfrac{x^2}{2} f(x) + C$

(E) $-\dfrac{x^2}{2} f(x)\left(1 + \dfrac{x}{3}\right) + C$

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