CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\int \frac{x^3 + 5}{x^2}\, dx =$

(A) $1 - \frac{10}{x^3} + C$

(B) $\frac{3x}{4} + \frac{15}{x^2} + C$

(C) $\frac{x^2}{2} - \frac{5}{x} + C$

(D) $\frac{x^2}{2} - \frac{5}{3x^3} + C$

(E) $-\frac{x^3}{4} - 5 + C$

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2. What is the slope of the line tangent to the graph of $y = \ln(2x)$ at the point where $x = 4$?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) 4

3. $\lim\limits_{x \to 0} \dfrac{x^2}{1 - \cos x}$ is

(A) -2 (B) 0 (C) 1 (D) 2 (E) nonexistent

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4. $\displaystyle\int \frac{1}{x^2 - 7x + 10}\,dx =$

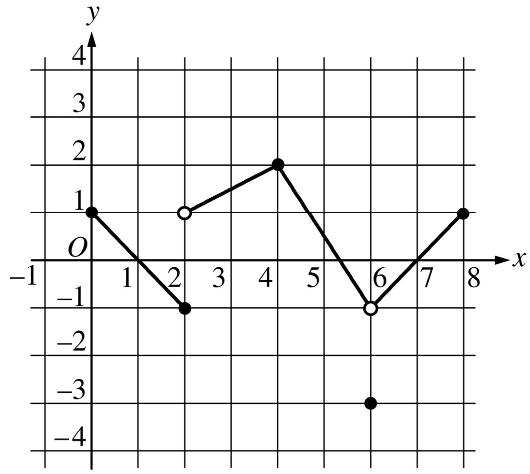
(A) $\ln\left|(x-2)(x-5)\right| + C$

(B) $\dfrac{1}{3}\ln\left|(x-2)(x-5)\right| + C$

(C) $\dfrac{1}{3}\ln\left|\dfrac{2x-7}{(x-2)(x-5)}\right| + C$

(D) $\dfrac{1}{3}\ln\left|\dfrac{x-2}{x-5}\right| + C$

(E) $\dfrac{1}{3}\ln\left|\dfrac{x-5}{x-2}\right| + C$



5. The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x) = f(2)$

II. $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

III. $\lim_{x \to 6} f(x) = f(6)$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

6. The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) The series diverges.

7. Let f be the function defined by $f(x) = \begin{cases} x^2 + 2 & \text{for } x \le 3, \\ 6x + k & \text{for } x > 3. \end{cases}$

If f is continuous at $x = 3$, what is the value of k ?

(A) -7 (B) 2 (C) 3 (D) 7 (E) There is no such value of k.

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8. $\int_0^1 x\sqrt{1 + 8x^2}\,dx =$

(A) $\frac{1}{24}$ (B) $\frac{13}{12}$ (C) $\frac{9}{8}$ (D) $\frac{52}{3}$ (E) 18

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9. The function f has a first derivative given by $f'(x) = x(x-3)^2(x+1)$. At what values of x does f have a relative maximum?

(A) -1 only (B) 0 only (C) -1 and 0 only (D) -1 and 3 only (E) -1, 0, and 3

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10. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

(A) $\frac{-2}{e^2 - 2e}$ (B) $\frac{-2}{e^2 + 2e}$ (C) $\frac{-2}{e + 2}$ (D) $\frac{e}{e + 2}$ (E) The series diverges.

$$f(x) = \begin{cases} 2x + 5 & \text{for } x < -1 \\ -x^2 + 6 & \text{for } x \geq -1 \end{cases}$$

11. If f is the function defined above, then $f'(-1)$ is

(A) -2 (B) 2 (C) 3 (D) 5 (E) nonexistent

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12. Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\,dx$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 120

13. A rectangular area is to be enclosed by a wall on one side and fencing on the other three sides. If 18 meters of fencing are used, what is the maximum area that can be enclosed?

(A) $\frac{9}{2}$ m^2 (B) $\frac{81}{4}$ m^2 (C) 27 m^2 (D) 40 m^2 (E) $\frac{81}{2}$ m^2

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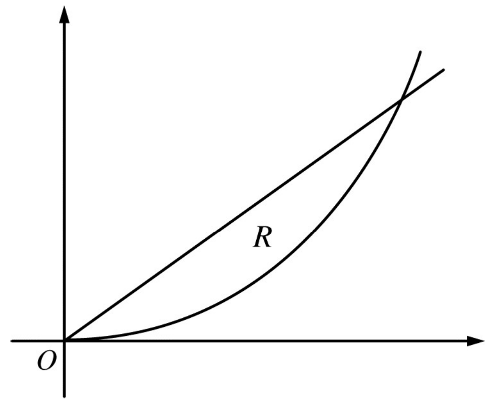
14. Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?

(A) 0 (B) $\frac{1}{4}$ (C) 6 (D) 24 (E) 144

15. Suppose $\ln x - \ln y = y - 4$, where y is a differentiable function of x and $y = 4$ when $x = 4$. What is the value of $\frac{dy}{dx}$ when $x = 4$?

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{17}{5}$



16. Let R be the region in the first quadrant that is bounded by the polar curves $r = \theta$ and $\theta = k$, where k is a constant, $0 < k < \frac{\pi}{2}$, as shown in the figure above. What is the area of R in terms of k ?

(A) $\frac{k^3}{6}$ (B) $\frac{k^3}{3}$ (C) $\frac{k^3}{2}$ (D) $\frac{k^2}{4}$ (E) $\frac{k^2}{2}$

17. Which of the following is the Maclaurin series for e^{3x} ?

(A) $1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots$

(B) $3 + 9x + \dfrac{27x^2}{2} + \dfrac{81x^3}{3!} + \dfrac{243x^4}{4!} + \cdots$

(C) $1 - 3x + \dfrac{9x^2}{2} - \dfrac{27x^3}{3!} + \dfrac{81x^4}{4!} - \cdots$

(D) $1 + 3x + \dfrac{3x^2}{2} + \dfrac{3x^3}{3!} + \dfrac{3x^4}{4!} + \cdots$

(E) $1 + 3x + \dfrac{9x^2}{2} + \dfrac{27x^3}{3!} + \dfrac{81x^4}{4!} + \cdots$

18. $\displaystyle\int_1^{\infty} \frac{x^2}{\left(x^3+2\right)^2}\,dx$ is

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$ (C) $\frac{1}{3}$ (D) 1 (E) divergent

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19. For what values of x does the graph of $y = 3x^5 + 10x^4$ have a point of inflection?

(A) $x = -\frac{8}{3}$ only

(B) $x = -2$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = -\frac{8}{3}$

(E) $x = 0$ and $x = -2$

20. If $f'(x) = \dfrac{(x-2)^3(x^2-4)}{16}$ and $g(x) = f(x^2-1)$, what is $g'(2)$?

(A) 2 (B) $\dfrac{5}{4}$ (C) $\dfrac{5}{8}$ (D) $\dfrac{5}{16}$ (E) 0

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x	1	3	5	7
$f(x)$	4	6	7	5
$f'(x)$	2	1	0	–1

21. The table above gives selected values for a differentiable function f and its first derivative. Using a left Riemann sum with 3 subintervals of equal length, which of the following is an approximation of the length of the graph of f on the interval $[1, 7]$?

(A) 6 (B) 34 (C) $2\sqrt{3} + 2\sqrt{2} + 2$ (D) $2\sqrt{5} + 2\sqrt{2} + 2$ (E) $2\sqrt{5} + 4\sqrt{2} + 2$

22. What is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 2^n}$?

(A) $1 < x < 5$

(B) $1 \le x < 5$

(C) $1 \le x \le 5$

(D) $2 < x < 4$

(E) $2 \le x \le 4$

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23. What is the particular solution to the differential equation $\frac{dy}{dx} = xy^2$ with the initial condition $y(2) = 1$?

(A) $y = e^{\frac{x^2}{2} - 2}$

(B) $y = e^{\frac{x^2}{2}}$

(C) $y = -\frac{2}{x^2}$

(D) $y = \frac{2}{6 - x^2}$

(E) $y = \frac{6 - x^2}{2}$

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24. Which of the following series converge?

I. $1 + (-1) + 1 + \cdots + (-1)^{n-1} + \cdots$

II. $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$

III. $1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-1}} + \cdots$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

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25. What is the slope of the line tangent to the polar curve $r = \cos\theta$ at the point where $\theta = \frac{\pi}{6}$?

(A) $-\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

26. For $x > 0$, $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{1}{1+t^2}\,dt =$

(A) $\frac{1}{2\sqrt{x}(1+x)}$ (B) $\frac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)}$ (C) $\frac{1}{1+x}$ (D) $\frac{\sqrt{x}}{1+x}$ (E) $\frac{1}{1+\sqrt{x}}$

27. What is the coefficient of x^2 in the Taylor series for $\sin^2 x$ about $x = 0$?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

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28. The function h is given by $h(x) = x^5 + 3x - 2$ and $h(1) = 2$. If h^{-1} is the inverse of h, what is the value of $\left(h^{-1}\right)'(2)$?

(A) $\frac{1}{83}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) 1 (E) 8

CALCULUS BC

SECTION I, Part B

Time—50 minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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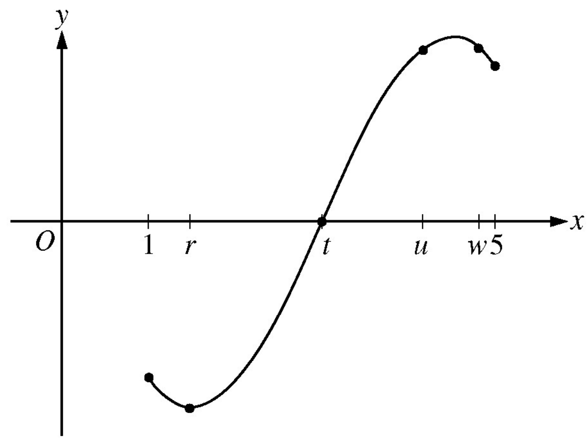

Graph of f

76. The figure above shows the graph of the differentiable function f for $1 \le x \le 5$. Which of the following could be the x-coordinate of a point at which the line tangent to the graph of f is parallel to the secant line through the points $(1, f(1))$ and $(5, f(5))$?

(A) r (B) t (C) u (D) w (E) There is no such point.

77. The number of antibodies y in a patient's bloodstream at time t is increasing according to a logistic differential equation. Which of the following could be the differential equation?

(A) $\dfrac{dy}{dt} = 0.025t$

(B) $\dfrac{dy}{dt} = 0.025t(5000 - t)$

(C) $\dfrac{dy}{dt} = 0.025y$

(D) $\dfrac{dy}{dt} = 0.025(5000 - y)$

(E) $\dfrac{dy}{dt} = 0.025y(5000 - y)$

GO ON TO THE NEXT PAGE.

78. What is the area of the region enclosed by the graphs of $y = \dfrac{1}{1 + x^2}$ and $y = x^2 - \dfrac{1}{3}$?

(A) 0.786 (B) 0.791 (C) 1.582 (D) 1.837 (E) 1.862

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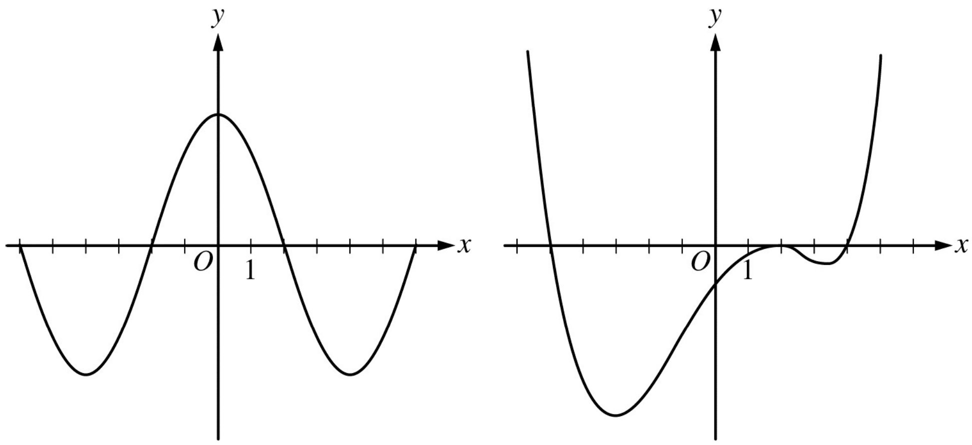
79. A vase has the shape obtained by revolving the curve $y = 2 + \sin x$ from $x = 0$ to $x = 5$ about the x-axis, where x and y are measured in inches. What is the volume, in cubic inches, of the vase?

(A) 10.716 (B) 25.501 (C) 33.666 (D) 71.113 (E) 80.115



Graph of f Graph of g

80. The graphs of two differentiable functions f and g are shown above. Given $p(x) = f(x)g(x)$, which of the following statements about $p'(-2)$ is true?

(A) $p'(-2) < 0$

(B) $p'(-2) = 0$

(C) $p'(-2) > 0$

(D) $p'(-2)$ is undefined.

(E) There is not enough information given to conclude anything about $p'(-2)$.

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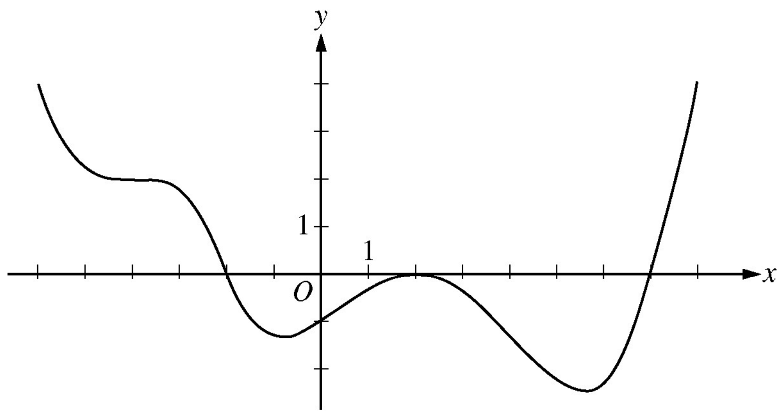
81. At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628



Graph of f'

82. The figure above shows the graph of f', the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection

(B) 1 relative minimum, 1 relative maximum, and 4 points of inflection

(C) 2 relative minima, 1 relative maximum, and 2 points of inflection

(D) 2 relative minima, 1 relative maximum, and 4 points of inflection

(E) 2 relative minima, 2 relative maxima, and 3 points of inflection

GO ON TO THE NEXT PAGE.

x	$f'(x)$
1	0.2
1.5	0.5
2	0.9

83. The table above gives values of f', the derivative of a function f. If $f(1) = 4$, what is the approximation to $f(2)$ obtained by using Euler's method with a step size of 0.5 ?

(A) 2.35

(B) 3.65

(C) 4.35

(D) 4.70

(E) 4.80

84. A sphere is expanding in such a way that the area of any circular cross section through the sphere's center is increasing at a constant rate of $2\,\text{cm}^2/\text{sec}$. At the instant when the radius of the sphere is 4 centimeters, what is the rate of change of the sphere's volume? (The volume V of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.)

(A) $8\,\text{cm}^3/\text{sec}$

(B) $16\,\text{cm}^3/\text{sec}$

(C) $8\pi\,\text{cm}^3/\text{sec}$

(D) $64\pi\,\text{cm}^3/\text{sec}$

(E) $128\pi\,\text{cm}^3/\text{sec}$

85. For $t \geq 0$, the components of the velocity of a particle moving in the xy-plane are given by the parametric equations $x'(t) = \frac{1}{t+1}$ and $y'(t) = ke^{kt}$, where k is a positive constant. The line $y = 4x + 3$ is parallel to the line tangent to the path of the particle at the point where $t = 2$. What is the value of k ?

(A) 0.072 (B) 0.433 (C) 0.495 (D) 0.803 (E) 0.828

t (hours)	0	1	2	3	4	5	6
$s(t)$ (miles)	0	25	55	92	150	210	275

86. The table above gives the distance $s(t)$, in miles, that a car has traveled at various times t, in hours, during a 6-hour trip. The graph of the function s is increasing and concave up. Based on the information, which of the following could be the velocity of the car, in miles per hour, at time $t = 3$?

(A) 37 (B) 49 (C) 58 (D) 65 (E) 92

87. If $0 < b_n < a_n$ for $n \geq 1$, which of the following must be true?

(A) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} b_n = 0$.

(C) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.

(D) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

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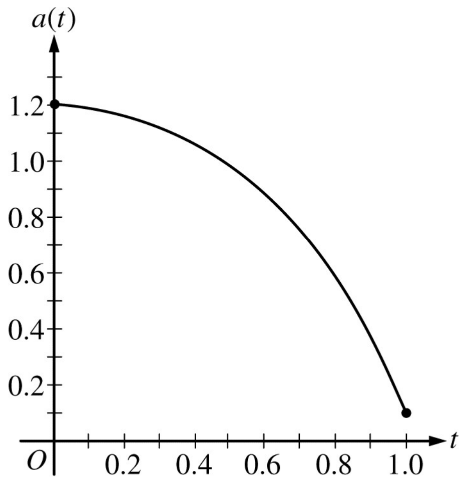

88. A particle moves along the x-axis so that its acceleration $a(t)$ is given by the graph above for all values of t where $0 \le t \le 1$. At time $t = 0$, the velocity of the particle is $-\frac{1}{2}$. Which of the following statements must be true?

(A) The particle passes through $x = 0$ for some t between $t = 0$ and $t = 1$.

(B) The velocity of the particle is 0 for some t between $t = 0$ and $t = 1$.

(C) The velocity of the particle is negative for all values of t between $t = 0$ and $t = 1$.

(D) The velocity of the particle is positive for all values of t between $t = 0$ and $t = 1$.

(E) The velocity of the particle is less than $-\frac{1}{2}$ for all values of t between $t = 0$ and $t = 1$.

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89. The function f is given by $f(x) = \int_1^x \sqrt{t^3 + 2}\, dt$. What is the average rate of change of f over the interval $[0, 3]$?

(A) 1.324 (B) 1.497 (C) 1.696 (D) 2.266 (E) 2.694

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90. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $(0, 0.177)$ and $(1.256, \infty)$

(B) $(0, 1.256)$ only

(C) $(0, 2.057)$ only

(D) $(0.177, 1.256)$ only

(E) $(0.177, 1.256)$ and $(2.057, \infty)$

91. Line ℓ is tangent to the graph of $y = \cos x$ at the point $(k, \cos k)$, where $0 < k < \pi$. For what value of k does line ℓ pass through the origin?

(A) 0.860

(B) 1.571

(C) 2.356

(D) 2.798

(E) There is no such value of k.

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x	2	4
$f(x)$	7	13
$g(x)$	2	9
$g'(x)$	1	7
$g''(x)$	5	8

92. The table above gives selected values of twice-differentiable functions f and g, as well as the first two derivatives of g. If $f'(x) = 3$ for all values of x, what is the value of $\int_2^4 f(x)g''(x)\,dx$?

(A) 63 (B) 69 (C) 78 (D) 84 (E) 103