CALCULUS BC FINAL EXAM SEMESTER 1 REVIEW

PART I NON-CALCULATOR: MULTIPLE-CHOICE

NO calculator may be used on this part of the review.

1. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

(A) $2x - 3y = 0$

(B) $4x - 5y = 2$

(C) $4x - y = 10$

(D) $5x - 4y = 7$

(E) $5x - 2y = 13$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 + 2t}{2t + 2}$ $\quad x(1) = 3$ $\quad y(1) = 2$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{5}{4}$

$y - 2 = \frac{5}{4}(x - 3)$

$y - 2 = \frac{5}{4}x - \frac{15}{4}$

$4y - 8 = 5x - 15$

$7 = 5x - 4y$

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2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

(A) $-\frac{3x+2y}{y^2}$

(B) $-\frac{3x+y}{x+y}$

(C) $\frac{1-3x-y}{x+y}$

(D) $-\frac{3x}{1+y}$

(E) $-\frac{3x}{x+y}$

$6x + y(2) + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x + 2y) = -6x - 2y$

$\frac{dy}{dx} = \frac{-2(3x+y)}{2(x+y)}$

$= -\frac{3x+y}{x+y}$

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x	–1.0	–0.5	0.0	0.5	1.0	1.5	2.0
$g'(x)$	2	4	3	1	0	–3	–6

3. The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

(A) –6.5

(B) –1.5

(C) 1.5

(D) 2.5

(E) 3

$g(-1) = -2$

$g(0.5) = -2 + 2(1.5)$

$= -2 + 3$

$= 1$

$g(2) = 1 + 1(1.5)$

$= 2.5$

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4. If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}f(h(x)) =$ $f'(h(x)) \cdot h'(x)$

$= f'(x^2) \cdot 2x$

$= g(x^2) \cdot 2x$

$= 2x\, g(x^2)$

(A) $g(x^2)$

(B) $2xg(x)$

(C) $g'(x)$

(D) $2xg(x^2)$

(E) $x^2g(x^2)$

5. If F' is a continuous function for all real x, then

$$\lim_{h\to 0} \frac{1}{h}\int_a^{a+h} F'(x)\,dx \text{ is}$$

(A) 0

(B) $F(0)$

(C) $F(a)$

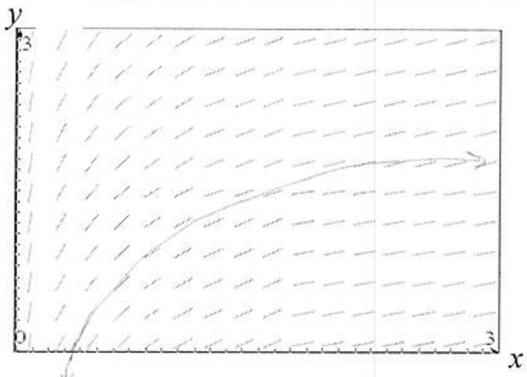
(D) $F'(0)$

(E) $\boxed{F'(a)}$

$\lim_{h\to 0} \frac{1}{h}\left[F(x) \Big|_a^{a+h} \right]$

$\lim_{h\to 0} \frac{1}{h}\left(F(a+h)-F(a)\right)$

$\lim_{h\to 0} \frac{F(a+h)-F(a)}{h}$ definition of derivative @ $x=a$ $F'(a)$



as $x \to \infty$, $\frac{dy}{dx} \to 0$

6. The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ $\frac{dy}{dx} = 2x$

(B) $y = e^x$ $\frac{dy}{dx} = e^x$

(C) $y = e^{-x}$ $\frac{dy}{dx} = e^{-x}(-1)$

(D) $y = \cos x$ $\frac{dy}{dx} = -\sin x$

(E) $\boxed{y = \ln x}$ $\frac{dy}{dx} = \frac{1}{x}$ as $x \to \infty$, $\frac{dy}{dx} \to 0$

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7. $\int_0^3 \frac{dx}{(1-x)^2}$ is

(A) $\boxed{-\frac{3}{2}}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

(E) None of these

$u = 1-x$

$\frac{du}{dx} = -1$

$-du = dx$

$u(0) = 1$

$u(3) = -2$

$\int_1^{-2} u^{-2} \cdot -du = -\int_1^{-2} u^{-2}\,du$

$= u^{-1} \Big|_1^{-2}$

$= \frac{1}{u} \Big|_1^{-2}$

$= -\frac{1}{2} - \left(\frac{1}{1}\right)$

$= -\frac{3}{2}$

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8. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0,k]$, then $k =$

(A) 3

(B) $3^{1/2}$

(C) $18^{1/3}$

(D) $36^{1/4}$

(E) $\boxed{36^{1/3}}$

avg value $= \frac{1}{k-0}\int_0^k x^3\,dx$

$9 = \frac{1}{k}\left(\frac{1}{4}x^4\right)\Big|_0^k$

$9 = \frac{1}{4k}\left(k^4 - 0^4\right)$

$9 = \frac{k^3}{4}$

$36 = k^3 \longrightarrow \sqrt[3]{36} = k$

9. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

$\boxed{\text{(C)}}$ $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^x$

$dy = y\sec^2 x\, dx$

$\int \frac{1}{y}\, dy = \int \sec^2 x\, dx$

$\ln|y| = \tan x + C$

$\ln 5 = \tan 0 + C$

$\ln 5 = C$

$e^{\ln|y|} = e^{\tan x + \ln 5}$

$|y| = e^{\tan x} \cdot e^{\ln 5}$

$|y| = 5e^{\tan x}$

$y = 5e^{\tan x}$ (b/c $y>0$) ☺

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10. Determine the value of c so that $f(x)$ continuous on the entire real line when $f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$

(A) 0

(B) -2

(C) 1

$\boxed{\text{(D)}}$ $-\frac{1}{2}$

(E) None of these

$\lim_{x \to 2^-} (x+3) = \lim_{x \to 2^+} (cx+6)$

$2 + 3 = 2c + 6$

$5 = 2c + 6$

$-1 = 2c$

$-\frac{1}{2} = c$

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11. Find $\frac{dy}{dx}$ if: $x^2 + 3xy + y^3 = 10$

$\boxed{\text{(A)}}$ $-\frac{2x+3y}{3x+3y^2}$

(B) $\frac{2x-3y}{3x+3y^2}$

(C) $-\frac{x+y}{x+y^2}$

(D) $\frac{x-y}{x+y^2}$

(E) None of these

$2x + y(3) + 3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3x + 3y^2) = -2x - 3y$

$\frac{dy}{dx} = -\frac{2x+3y}{3x+3y^2}$

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12. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top moving down the wall when the base of the ladder is 7 feet.

$\boxed{\text{(A)}}$ $\frac{7}{12}$ ft / ~~min~~ sec

(B) $-\frac{7}{12}$ ft / ~~min~~ sec

(C) ~~10π~~ $12/7$ ft / ~~min~~ sec

(D) ~~9π~~ $-12/7$ ft / ~~min~~ sec

(E) None of these

$l = 25$ ft

l is constant

$\frac{dx}{dt} = 2$ ft/sec

$x = 7$ ft

$\frac{dy}{dt} = ?$

$x^2 + y^2 = l^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(7)(2) + 2(24)\frac{dy}{dt} = 0$

$2(24)\frac{dy}{dt} = -2(7)(2)$

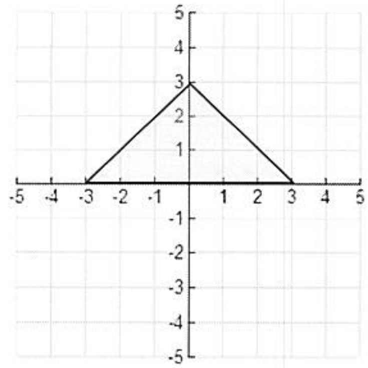
$\frac{dy}{dt} = \frac{-2(7)(2)}{2(24)}$

$= \frac{-7}{12}$ ft/sec

Ladder moving down @ $\frac{7}{12}$ ft/sec

13. Set up a definite integral that yields the area of the region.

$f(x) = 3 - |x|$



area of shaded region $= \int_{-3}^{3} (3 - |x|)\, dx$

(A) $\int_3^3 (3 - x)\, dx$

(B) $\int_0^3 |x|\, dx$

(C) $\int_3^{-3} (3 - |x|) dx$

(D) $\int_{-3}^{3} (3 - |x|) dx$

(E) $\int_3^0 |x|\, dx$

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14. A particle moves on a plane curve so that at any time $t > 0$, its position can be represented by: $x(t) = t^3 - t$ and $y(t) = (2t-1)^3$. The acceleration vector of the particle at $t = 1$ is:

$\langle x''(t), y''(t) \rangle$

(A) (0, 1)

(B) (2, 3)

(C) (2, 6)

(D) (6, 12)

(E) (6, 24)

$v(t) = \langle 3t^2 - 1, 3(2t-1)^2(2) \rangle$

$= \langle 3t^2 - 1, 6(2t-1)^2 \rangle$

$a(t) = \langle 6t, 12(2t-1)(2) \rangle$

$= \langle 6t, 24(2t-1) \rangle$

$a(1) = \langle 6, 24 \rangle$

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15. If $f(x) = \sqrt{e^x}$, then $f'(\ln 2) =$

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) 1

(E) $\sqrt{2}$

$f(x) = (e^x)^{\frac{1}{2}}$

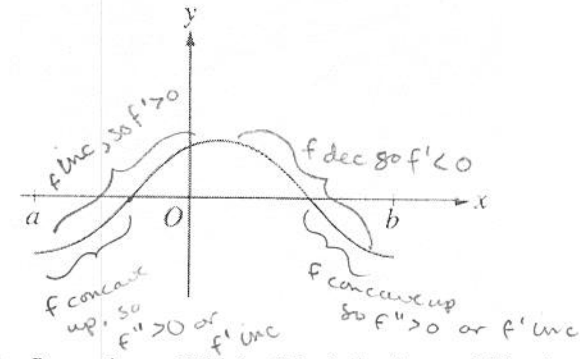
$f'(x) = \frac{1}{2}(e^x)^{-1/2} \cdot e^x$

$f'(\ln 2) = \frac{1}{2}(e^{\ln 2})^{-1/2} \cdot e^{\ln 2}$

$= \frac{1}{2}(2)^{-1/2} \cdot 2$

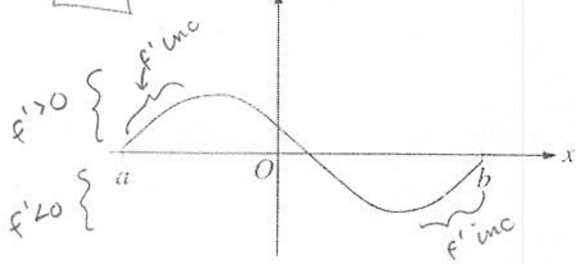
$= 2^{-1/2}$

$= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$



16. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

$\boxed{\text{(A)}}$



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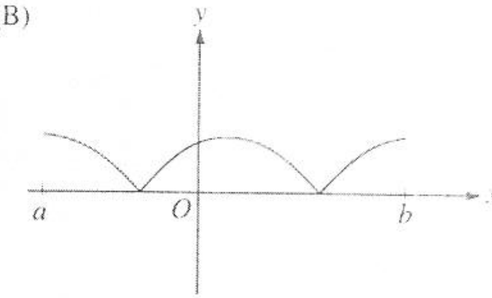

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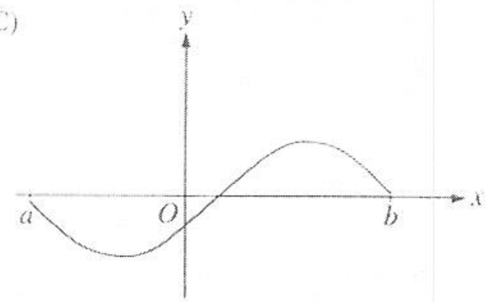
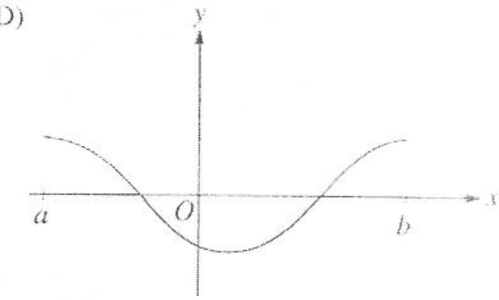
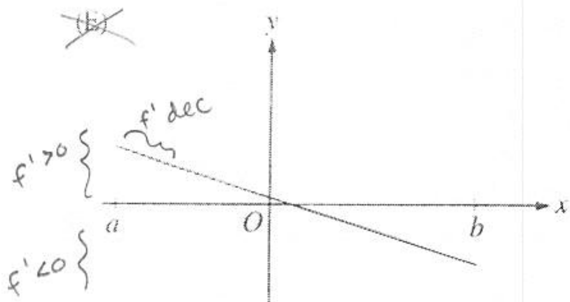


(D)



~~(E)~~



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17. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0, 2]?

$\boxed{\text{(A)}}$ $\frac{26}{9}$

(B) $\frac{52}{9}$

(C) $\frac{26}{3}$

(D) $\frac{52}{3}$

(E) 24

avg value $= \frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx$

$= \frac{1}{2}\int_0^2 x^2(x^3+1)^{1/2}\,dx$

$= \frac{1}{2}\int_1^9 \cancel{x^2}\cdot u^{1/2}\cdot\frac{du}{3\cancel{x^2}}$

$= \frac{1}{6}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^9$

$= \frac{1}{9}\left(9^{3/2} - 1^{3/2}\right) = \frac{1}{9}(27-1) = \frac{26}{9}$

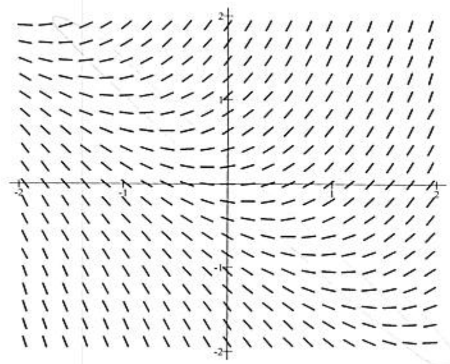
$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3x^2} = dx$

$u(0) = 1$

$u(2) = 9$



depends on $x + y$

$\rightarrow$ when $x = -y$, $\frac{dy}{dx} = 0$

18. Shown above is a slope field for which of the following differential equations?

~~(A)~~ $\frac{dy}{dx} = 1 + x$

~~(B)~~ $\frac{dy}{dx} = x^2$

$\boxed{\text{(C)}}$ $\frac{dy}{dx} = x + y$

when $x = -y$,
$\frac{dy}{dx} = -y + y = 0$

~~(D)~~ $\frac{dy}{dx} = \frac{x}{y}$

~~(E)~~ $\frac{dy}{dx} = \ln y$

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19. If $F(x) = \int_0^{2x} \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) -3

(B) -2

(C) 3

$\boxed{\text{(D)}}$ 6

(E) 18

$F'(x) = \frac{d}{dx} \int_0^{2x} \sqrt{t^3+1}\, dt$

$F'(x) = \sqrt{(2x)^3 + 1} \cdot 2$

$F'(2) = \sqrt{9} \cdot 2$

$F'(2) = 3 \cdot 2$

$F'(2) = 6$

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20. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

$\boxed{\text{(A)}}$ $\frac{3x-3}{\sqrt{2x-3}}$

(B) $\frac{x}{\sqrt{2x-3}}$

(C) $\frac{-x+3}{\sqrt{2x-3}}$

(D) $\frac{1}{\sqrt{2x-3}}$

(E) $\frac{5x-6}{\sqrt{2x-3}}$

$f'(x) = (2x-3)^{1/2}(1) + x\left(\frac{1}{2}(2x-3)^{-1/2} \cdot 2\right)$

$= (2x-3)^{1/2} + x(2x-3)^{-1/2}$

$= (2x-3)^{1/2} + \frac{x}{(2x-3)^{1/2}}$

$= \frac{2x-3+x}{(2x-3)^{1/2}}$

$= \frac{3x-3}{\sqrt{2x-3}}$

21. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval [-2, 4] occurs at x =

(A) -2 (B) 0 (C) 1 (D) 2 $\boxed{\text{(E)}}$ 4

$f'(x) = 3x^2 - 6x$
$0 = 3x(x-2)$
$x = 0, x = 2$ crit #s
$f(0) = 12$, $f(2) = 8$, $f(-2) = -8$
$f(4) = 28$

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22. Integrate: $\int \cancel{\frac{e^{x/2}}{2}}\,dx$ $\int x e^{x/2}\,dx$

$u = x$ $du = dx$
$dv = e^{x/2}\,dx$ $v = 2e^{x/2}$

$x \cdot 2e^{x/2} - \int 2e^{x/2}\,dx$
$2xe^{x/2} - 4e^{x/2} + C$

(A) $\cancel{e^x + C}$ $\boxed{\text{(B)}}$ $\cancel{e^{\frac{-x}{2}} + C}$ (C) $e^{\frac{x}{2}} + C$ (D) $\cancel{2e^{\frac{x}{2}} + C}$ (E) $\cancel{e^x + C}$

---

23. If $f(x) = \sin^{-1} x$, then $f'\left(\frac{1}{2}\right) =$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$
$f'(\tfrac{1}{2}) = \frac{1}{\sqrt{1-(\frac{1}{2})^2}} \to \frac{1}{\sqrt{1-1/4}} = \frac{1}{\sqrt{3/4}} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$
$= \frac{2\sqrt{3}}{3}$

$\boxed{\text{(A)}}$ $\frac{2\sqrt{3}}{3}$ (B) $\frac{4}{5}$ (C) $-\frac{4}{5}$ (D) $-\frac{2\sqrt{3}}{3}$ (E) $\frac{\pi}{2}$

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24. A spherical balloon is inflated with gas at the rate of 800 cubic cm per minute. How fast is the radius of the balloon increasing at the instant the radius is 30 cm?

(HINT: $V = \frac{4}{3}\pi r^3$)

$\frac{dV}{dt} = 800\,cm^3/min$
$r = 30\,cm$
$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$
$800 = 4\pi(30)^2 \frac{dr}{dt}$
$\frac{800}{4\pi \cdot 900} = \frac{dr}{dt}$
$\frac{8}{36\pi} = \frac{dr}{dt}$
$\frac{2}{9\pi}\,cm/min = \frac{dr}{dt}$

$\boxed{\text{(A)}}$ $\frac{2}{9\pi}$ cm/min (B) $\frac{9}{2\pi}$ cm/min (C) 9 cm/min

(D) 2cm/min (E) None of these

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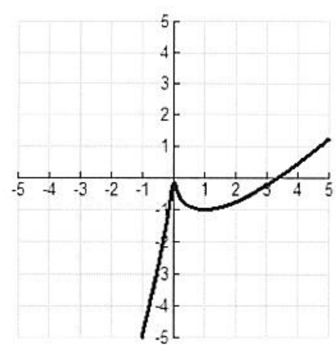
25. The graph shown represents $y = f(x)$. Which of the following is Not True?



True (A) f is continuous on the interval [-1,1] no jumps, holes or asymptotes on [-1,1]

True (B) $\lim_{x \to 0} f(x) = f(0)$ no discont @ x = 0

True (C) f is concave up on $(0, \infty)$ f curved up on $(0,\infty)$

$\boxed{\text{(D)}}$ f has minimum at (-1,-5) and maximum (1,-1) on the interval [-1,3]

(E) All are true

↳ FALSE maximum @ (0,0)

26. If f is continuous on $[-2,4]$ and $f(-2)=5, f(0)=-3,$ *and* $f(4)=711$, then according to the Intermediate Value Theorem, how many zeroes are guaranteed on the closed interval $[-2, 4]$?

(A) none (B) one $\boxed{\text{(C)}}$ two (D) three (E) four

f + − +
(-2) (0) (4)
b/c f changes signs twice, by IVT, 2 zeroes on [-2, 4]

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27. $\lim_{h\to 0}\dfrac{\cos\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}\right)}{h}=?$

definition of derivative
$f'(a)$ where $f(x) = \cos x$
@ $a = \pi/2$

$f'(x) = -\sin x$
$f'(\pi/2) = -\sin \pi/2$
$= -1$

$\boxed{\text{(A)}}$ -1

(B) 0

(C) 1

(D) $\cos\left(\frac{\pi}{2}+h\right)$

(E) undefined

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28. $\int x\sqrt{3x}dx =$ $\quad \sqrt{3}\int x\sqrt{x}\,dx = \sqrt{3}\int x^{3/2}\,dx$

$= \sqrt{3}\left(\frac{2}{5}x^{5/2}\right) + c$

$= \frac{2\sqrt{3}}{5}x^{5/2} + c$

$\boxed{\text{(A)}}$ $\dfrac{2\sqrt{3}}{5}x^{5/2}+C$

(B) $\dfrac{5\sqrt{3}}{2}x^{\frac{5}{2}}+C$

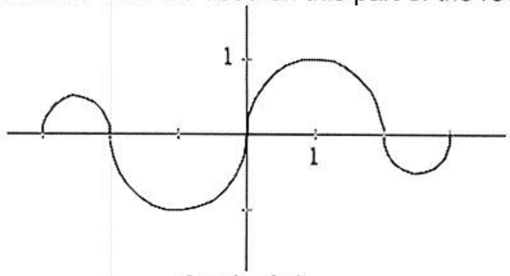
(C) $\dfrac{\sqrt{3}}{2}x^{\frac{1}{2}}+C$

(D) $2\sqrt{3x}+C$

(E) $\dfrac{5\sqrt{3}}{2}x^{\frac{3}{2}}+C$

PART II CALCULATOR: MULTIPLE-CHOICE

A calculator MAY be used on this part of the review



Graph of f

1. The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)dt$, where is $g(x)$ nonnegative?

$\boxed{\text{(A)}}$ $[-3, 3]$

(B) $[-3, 2] \cup [0,2]$ only

(C) $[0,3]$ only

(D) $[0,2]$ only

(E) $[-3, -2] \cup [0,3]$ only

$g(2) = \int_0^2 f(t)dt = \frac{1}{2}\pi(1)^2 = \frac{1}{2}\pi$

$g(3) = \frac{1}{2}\pi - \frac{1}{4}\pi = \frac{1}{4}\pi$

$g(-2) = \int_0^{-2} f(t)dt = -\int_{-2}^0 f(t)dt = -(-\frac{1}{2}\pi) = \frac{\pi}{2}$

$g(-3) = -(\frac{\pi}{4} - \frac{\pi}{2}) = -(-\frac{\pi}{4}) = \frac{\pi}{4}$

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2. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$. TRUE b/c if f diff'able, f cont.

(B) $\lim_{x\to a} f(x)$ exists. since f cont, limit exists

(C) $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists. def. of derivative @ $x = a$

(D) $f'(a)$ is defined. def of derivative @ $x = a$

$\boxed{\text{(E)}}$ $f''(a)$ is defined. no info about f'', so could be FALSE

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3. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) −3.268

$\boxed{\text{(B)}}$ −1.585

(C) 1.732

(D) 6.585

(E) 11.585

$g(x) = g(3) + \int_3^x \sqrt{t^3+2}\, dt$ ⇝ b/c $\int_3^x f(t)dt = g(x)\Big|_3^x$; $\int_3^x f(t)dt = g(x) - g(3)$

$g(1) = g(3) + \int_3^1 \sqrt{x^3+2}\, dx$

$= -1.58458$

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4. Let g be the function given by $g(x) = \int_1^x 100(t^2 - 3t + 2)e^{-t^2}\, dt$. Which of the following statements about g must be true?

~~I.~~ g is increasing on $(1, 2)$. ⇒ $g' < 0$ so g dec

II. g is increasing on $(2, 3)$. ⇒ $g' > 0$

~~III.~~ $g(3) > 0$

$g'(x) = 100(x^2 - 3x + 2)e^{-x^2}$

$g'(x) > 0$ on $(2,3)$

$g(3) = \int_1^3 100(t^2 - 3t + 2)e^{-t^2}\, dt$

$= -1.942$

g': + (0) + (1) − (2) +

(A) I only

$\boxed{\text{(B)}}$ II only

(C) III only

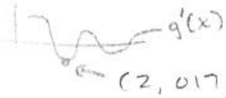
(D) II and III only

(E) I, II, and III

5. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$. For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

(A) 0.949
$\boxed{\text{(B)}}$ 2.017
(C) 3.106
(D) 5.965
(E) 8.000

g dec most rapidly when
$g'(x)$ is lowest pt.



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6. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$

$\boxed{\text{(A)}}$ -3
(B) 0
(C) 3
(D) -3 and 3
(E) -3, 0, and 3

$\frac{1}{3}x^3 \Big|_{-3}^{k} = 0$

$\frac{1}{3}k^3 - \left(\frac{1}{3}(-3)^3\right) = 0$

$\frac{1}{3}k^3 + 9 = 0$

$\frac{1}{3}k^3 = -9$ → $k^3 = -27$

$k = -3$

---

7. If f is the function defined by $f(x) = \sqrt[3]{5x + x^2}$ and g is an antiderivative of f such that $g(5) = 8$, then $g(1) \approx$

(A) 3.375
(B) 2.665
(C) 1.817
$\boxed{\text{(D)}}$ -3.375
(E) -2.665

$g(1) = g(5) + \int_5^1 \sqrt[3]{5x + x^2}\,dx$

$= 8 + -11.3746$

$= -3.375$

---

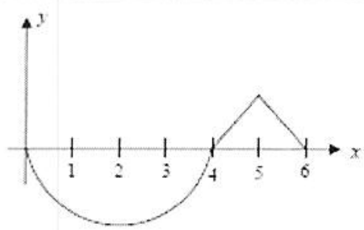
8. Let f be the function given by $f(x) = \tan x$ and let g be the function given by $g(x) = x^3$. At what value of x in the interval $0 \le x \le \pi$ do the graphs of f and g have parallel tangent lines?

↳ $g'(x) = f'(x)$

(A) 0
(B) 0.75
$\boxed{\text{(C)}}$ 1.883
(D) 1.697
(E) 10.63

$\sec^2 x = 3x^2$

$x = 1.883$



$g(0) = \int_4^0 f(t)\,dt = -\int_0^4 f(t)\,dt$

$= -\left(\frac{1}{2}\pi(2)^2\right)$

$= -2\pi$

9. The graph of f given above consists of two line segments and a semicircle. If $g(x) = \int_4^x f(t)\,dt$, then $g(0) =$

(A) -4π $\boxed{\text{(B)}}$ -2π (C) $-\pi$ (D) 2π (E) cannot be determined

10. The graph of the function $y = x^5 - x^2 + \sin x$ has a point of inflection at $x =$

y'' changes signs

(A) 0.324 $\boxed{\text{(B)}}$ 0.499 (C) 0.506 (D) 0.611 (E) 0.704

$y'' = 0$ @ $x = 0.499$

---

11. If $f(x) = 2g(x) - 1$ for $1 \le x \le 3$, then $\int_1^3 (f(x) + g(x))\,dx =$ $\int_1^3 (2g(x) - 1 + g(x))\,dx = \int_1^3 (3g(x) - 1)\,dx$

$= 3\int_1^3 g(x)dx - \int_1^3 dx$

$= 3\int_1^3 g(x)dx - (x)\Big|_1^3$

$= 3\int_1^3 g(x)dx - (3-1)$ ← $3\int_1^3 g(x)dx - 2$

(A) $\int_1^3 g(x)dx - 2$ (C) $2\int_1^3 g(x)dx - 2$ (E) $3\int_1^3 g(x)dx - x$

$\boxed{\text{(B)}}$ $3\int_1^3 g(x)dx - 2$ (D) $2\int_1^3 g(x)dx - 1$

---

12. Let h be the function defined by $h(x) = \cos 3x + \ln 4x$. What is the least value of x at which the graph of h changes concavity?

→ h'' changes signs, graph h'' & get x-value when h'' changes from neg to pos or pos to neg.

(A) 1.555 $\boxed{\text{(B)}}$ 0.621 (C) 0.371 (D) 0.096 (E) 0.004

---

13. A particle moves on the x-axis with velocity given by $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \le t \le 3$. How many times does the particle change direction as t increases from -3 to 3?

→ when velocity changes signs

(A) zero (B) one $\boxed{\text{(C)}}$ two (D) three (E) four

v(t) -3 3 v(t) changes signs twice

$x'(t) = 2t$ $y'(t) = \cos(4t)\cdot 4$

14. A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = \sqrt{(2(3))^2 + (4\cos(4\cdot 3))^2}$

(A) 2.909 (B) 3.062 $\boxed{\text{(C)}}$ 6.884 (D) 9.016 (E) 47.393

$= 6.884$

---

15. The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

→ highest positive velocity

v(t) max @ $t = .316$

$h(.316) = 10.263$

(A) 2.545 meters $\boxed{\text{(B)}}$ 10.263 meters (C) 34.125 meters (D) 54.889 meters (E) 89.005 meters

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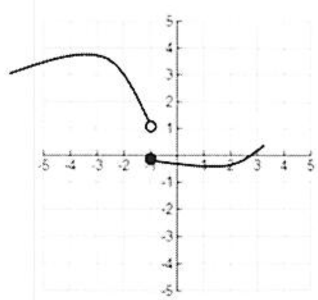


$\lim_{x\to -1^+} \sin(f(x))$

$= \sin\left(\lim_{x\to -1^+} f(x)\right)$

$= \sin(0)$

$= 0$

16. The graph of the function f is shown in the figure above. The value of $\lim_{x\to -1^+} \sin(f(x))$ is

(A) 0.909 (B) 0.540 (C) .017 $\boxed{\text{(D)}}$ 0 (E) nonexistent

---

17. Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects during the time interval $7 \le t \le 14$?

$\int_7^{14} \left(\frac{100e^{-.1t}}{2-e^{-3t}}\right)dt$

$\boxed{\text{(A)}}$ 125 (B) 100 (C) 88 (D) 50 (E) 12

PART III CALCULATOR: FREE-RESPONSE

A calculator MAY be used on this part of the review.

1. The function f is continuous on the closed interval [0, 10] and has values that are given in the table below. Using five equal subintervals, what is the left sum, right sum, midpoint sum, and trapezoidal approximations of $\int_0^{10} f(x)\,dx$?

$\Delta x = \frac{10-0}{5} = 2$

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	-4.5	-12	-20.5	-30

left sum $= 2(20+18+12+2+-12) = 80$

midpt sum $= 2(19.5+15.5+7.5+-4.5+-20.5) = 35$

right sum $= 2(-30+-12+2+12+18) = -20$

trap sum $= \frac{1}{2}(2)(20+2(18)+2(12)+2(2)+2(-12)+-30)$
$= 30$

2. If $\frac{dy}{dx} = \frac{x}{\sqrt{9+x^2}}$ and $y = 5$, when $x = 4$, find the equation of the curve.

$\int dy = \int \frac{x}{\sqrt{9+x^2}}\,dx$

$u = 9+x^2$

$\frac{du}{2x} = dx$

$y = \int \frac{x}{u^{1/2}} \cdot \frac{du}{2x}$

$y = \frac{1}{2}\int u^{-1/2}\,du$

$y = u^{1/2} + C$

$y = (9+x^2)^{1/2} + C$

$5 = (9+4^2)^{1/2} + C$

$5 = (25)^{1/2} + C$

$5 = 5 + C$

$0 = C$

$y = \sqrt{9+x^2}$

PART IV NON-CALCULATOR: FREE-RESPONSE

<u>NO</u> calculator may be used on this part of the review.

1. The function $f(x) = x^3 + ax^2 + bx + c$ has a relative maximum at (–3, 25) and a point of inflection at $x = -1$. Find a, b, and c. (see next page)

2. Water is being pumped into a conical reservoir (vertex down) at the constant rate of 10π ft^3/min. If the reservoir has a radius of 4 ft and is 12 ft deep, how fast is the water rising when the water is 6 ft deep? (see next page)

3. For what values of t does the curve given by the parametric equations

$x(t) = t^3 - t^2 - 1$

$y(t) = t^4 + 2t^2 - 8t$

have a vertical tangent?

$\frac{y'(t)}{x'(t)} = \frac{dy}{dx} = \frac{4t^3+4t-8}{3t^2-2t}$

when $\frac{dy}{dx} = \frac{1}{0}$ (undefined)

$3t^2 - 2t = 0$

$t(3t-2) = 0 \rightarrow t = 0,\ t = 2/3$

4. State the set of values for which $f(x) = (x-2)(x-3)^2$ is BOTH increasing and concave up. (See next page)

PART III CALCULATOR: FREE-RESPONSE

A calculator MAY be used on this part of the review.

1. The function f is continuous on the closed interval [0, 10] and has values that are given in the table below. Using five equal subintervals, what is the left sum, right sum, midpoint sum, and trapezoidal approximations of $\int_0^{10} f(x)\,dx$?

$\Delta x = \frac{10-0}{5} = 2$

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	-4.5	-12	-20.5	-30

left sum $= 2(20+18+12+2+-12) = 80$ right sum $= 2(-30+-12+2+12+18) = -20$

midpt sum $= 2(19.5+15.5+7.5+-4.5+-20.5) = 35$

2. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$. Show that the number of mosquitoes is increasing at time $t = 6$. At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate?

$R(6) = 5\sqrt{6}\cos(6/5) = 4.438 > 0$, $R(6) > 0$, so # mosquitoes increasing @ $t = 6$

$R'(6) = -1.913 < 0$, ∴ # mosquitoes is increasing @ a decreasing rate @ $t = 6$

PART IV NON-CALCULATOR: FREE-RESPONSE

NO calculator may be used on this part of the review.

1. The function $f(x) = x^3 + ax^2 + bx + c$ has a relative maximum at (–3, 25) and a point of inflection at $x = -1$. Find a, b, and c.

$f'(x)$ changes from pos to neg.

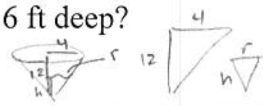
$f''(x)$ changes signs

2. Water is being pumped into a conical reservoir (vertex down) at the constant rate of 10π ft^3/min. If the reservoir has a radius of 4 ft and is 12 ft deep, how fast is the water rising when the water is 6 ft deep?

$\frac{dV}{dt} = 10\pi\ ft^3/min$

$h = 6 ft$

$\frac{dh}{dt} = ?$



$\frac{4}{r} = \frac{12}{h}$

$4h = 12r$

$\frac{1}{3}h = r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$10\pi = \frac{1}{9}\pi(6)^2 \frac{dh}{dt}$

$10\pi = 4\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{5}{2}$ ft/min

3. State the set of values for which $f(x) = (x-2)(x-3)^2$ is BOTH increasing and concave up.

$f' > 0$ $f'' > 0$

① $f'(x) = 3x^2 + 2ax + b$

$f'(-3) = 3(-3)^2 + 2a(-3) + b$

$0 = 27 - 6a + b$

$0 = 27 - 6(3) + b$

$0 = 27 - 18 + b$

$0 = 9 + b$

$\boxed{-9 = b}$

$f''(x) = 6x + 2a$

$f''(-1) = 6(-1) + 2a$

$0 = -6 + 2a$

$6 = 2a$

$\boxed{3 = a}$

$f(-3) = (-3)^3 + 3(-3)^2 + -9(-3) + c$

$25 = -27 + 27 + 27 + c$

$25 = 27 + c$

③ $f'(x) = (x-3)^2(1) + (x-2)[2(x-3)(1)]$

$= (x-3)^2 + 2(x-2)(x-3)$

$= (x-3)[x-3+2(x-2)]$

$0 = (x-3)(3x-7)$

$x = 3,\ x = 7/3$

f': + | − | + at 0, 7/3, 2.5, 3, 5

$f'(x) = 3x^2 - 16x + 21$

$f''(x) = 6x - 16$

$0 = 2(3x-8)$

$x = 8/3$

f'': − | + at 0, 8/3, 5