CollegeBoard
Advanced Placement
Program

AP® Calculus BC
2010 Free-Response Questions

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

rate accumulate

rate: remove

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

$$\text{accumulated Snow} = \int_0^6 7te^{\cos t}\,dt$$

1pt - integral

$$= 142.275 \text{ ft}^3$$

1pt - answer

(b) Find the rate of change of the volume of snow on the driveway at $\boxed{\text{8 A.M.}}$

difference in rate of snow

$t=8$

$$\text{rate of volume of snow} = \text{rate accumulated} - \text{rate removed}$$

$$= f(8) - g(8)$$

$$= 48.417 - 108$$

$$= -59.583 \text{ ft}^3/\text{hr}$$

1pt - answer

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Continue problem 1 on page 5.

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(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

amount snow removed $= \int_0^x g(t)\,dt$

$$h(t) = \begin{cases} \int_0^0 0\,dt & 0 \le t < 6 \\ h(6) + \int_6^t 125\,dx & 6 \le t < 7 \\ h(7) + \int_7^t 108\,dx & 7 < t \le 9 \end{cases} = \begin{cases} 0 & 0 \le t < 6 \\ 125t - 750 & 6 \le t < 7 \\ 108t - 631 & 7 < t \le 9 \end{cases}$$

1pt - $h(t)$ for $0 \le t < 6$

1pt - $h(t)$ for $6 < t \le 7$

1pt - $h(t)$ for $7 < t \le 9$

$h(6) + \int_6^t 125\,dx$

$= 0 + (125x)\Big|_6^t$

$= 0 + 125t - 125(6)$

$= 125t - 750$

$h(7) + \int_7^t 108\,dx$

$= 125 + (108x)\Big|_7^t$

$= 125 + 108t - 108(7)$

$= 108t - 631$

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

$\hookrightarrow \int$ rate accum $- \int$ rate remove

snow on driveway $= \int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt$

$= \int_0^9 f(t)\,dt - \left[\int_0^6 g(t)\,dt + \int_6^7 g(t)\,dt + \int_7^9 g(t)\,dt \right]$

$= 367.335 - [0 + 125 + 216]$

$= 26.335\ ft^3$

1pt - $\int_0^9 f(x)\,dt$

1pt - $\int_0^9 g(t)\,dt$ or $h(9)$

1pt - answer

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t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

↳ deposited

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

$$E'(6) = \frac{E(7)-E(5)}{7-5} \quad \frac{\text{hundreds of entries}}{\text{hrs}}$$

$$= \frac{21-13}{2}$$

$$= 4 \ \frac{\text{hundreds of entries}}{\text{hr}}$$

1pt - answer

avg value!

(b) Use a trapezoidal sum with the $\boxed{\text{four}}$ subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$.

Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

$$\frac{1}{8}\int_0^8 E(t)\,dt = \frac{1}{8}\left[\frac{1}{2}(0+4)(2) + \frac{1}{2}(4+13)(3) + \frac{1}{2}(13+21)(2) + \frac{1}{2}(21+23)(1)\right]$$

1pt - trap sum

$$\frac{1}{\cancel{hr}}\left(\frac{1}{2}(\text{hundred entries})(\cancel{hr})\right)$$

$$= 10.688 \text{ hundreds of entries}$$

1pt - approx

$\frac{1}{8}\int_0^8 E(t)\,dt$ means avg # of entries, in hundreds, deposited from $t=0$ to $t=8$ hrs (noon to 8pm)

1pt - meaning

Continue problem 2 on page 7.

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(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?

rate processed

processed time period

entries not processed = # deposited @ $t=12$ − # processed @ $t=12$

$= E(12) - \int_8^{12} P(t)\,dt$

$= 23 - 16$

$= 7$ hundred entries

1pt - integral

1pt - answer

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

process max
$P'(t)=0$
$P'(t)$ change pos to neg + check endpts

$P'(t) = 0$

@ $t = 9.184$, $t = 10.817$

P': + (8 to 9.184), − (9.184 to 10.817), + (10.817 to 12) ... ☺

rel. max @ $t = 9.184$ b/c P' changes from pos to neg @ $t = 9.184$

1pt - set $P'(t)=0$

1pt - crit #s

endpt → $P(8) = 0$

$P(9.184) = 5.089$

endpt → $P(12) = 8$

1pt - answer w/ reason

Entries processed most quickly @ $t = 12$

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3. A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds.

$\hookrightarrow |v|$

$\text{speed} = |v| = \sqrt{(x'(t))^2 + (y'(t))^2}$

$\text{speed @ } t=3 = \sqrt{(x'(3))^2 + (y'(3))^2}$

$= 2.828 \text{ m/sec}$

1pt - answer

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(b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

$\hookrightarrow \int |v(t)|\,dt$

$\text{Total distance} = \int_0^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= \int_0^4 \sqrt{(2t-4)^2 + (te^{t-3} - 1)^2}\,dt$

$= 11.588 \text{ meters}$

1pt - integral
1pt - answer

Continue problem 3 on page 9.

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(c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

$v(t) = x'(t)$ <0 or >0 $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$$

$dy/dt = 0$ and $dx/dt \neq 0$

$te^{t-3} - 1 = 0$ and $2t - 4 \neq 0$

$t = 2.208$

Particle moves right @ $t = 2.208$ b/c $x'(2.208) > 0$

1pt - $\frac{dy}{dx} = 0$

1pt - $t = 2.208$

1pt - answer w/ reason

(d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following. $\to x(t) = 5$

(i) The two values of t when that occurs

(ii) The slopes of the lines tangent to the particle's path at that point $\to dy/dx$

(iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$ $\to \int y'(t)dt$

(i) $x(t) = 5$

$t = 1$ and $t = 3$

1pt - $t = 1$ and $t = 3$

(ii) $\frac{dy}{dx}\Big|_{t=1} = \frac{dy/dt}{dx/dt}\Big|_{t=1} = 0.432$

$\frac{dy}{dx}\Big|_{t=3} = \frac{dy/dt}{dx/dt}\Big|_{t=3} = 1$

1pt - slopes

(iii) $y(1) = 3 + \frac{1}{e} + \int_2^1 (y'(t))\,dt$

1pt - $y(1)$

$y(1) = 4$

$y(3) = 3 + \frac{1}{e} + \int_2^3 (y'(t))\,dt = 4$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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