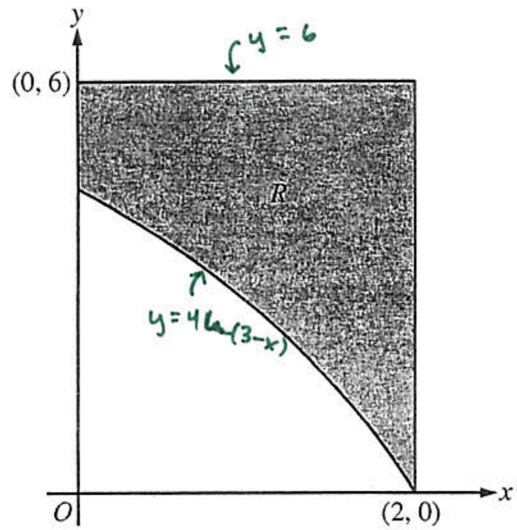
CollegeBoard
Advanced Placement
Program

AP® Calculus BC
2010 Free-Response Questions
Form B

The College Board

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R. top - bottom .. :)

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$$\text{Area of } R = \int_0^2 (6 - 4\ln(3-x))\,dx$$

$$= 6.817$$

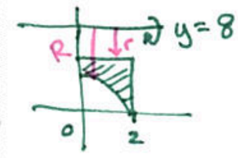
1pt - integrand

1pt - answer

1pt - correct limits in (a), (b), OR (c)

Continue problem 1 on page 5.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.



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outside radius inside radius

$$V = \pi \int_0^2 \left[\left(8 - 4\ln(3-x)\right)^2 - \left(8 - 6\right)^2 \right] dx$$

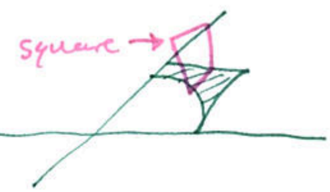
$$= 168.180$$

2pts - integrand

1pt - answer

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a $\boxed{\text{square}}$. Find the volume of the solid.

↳ work in x's



$$V = \int_0^2 \left(6 - 4\ln(3-x)\right)^2 dx$$

$$= 26.267$$

$$\text{Area} = (\text{side})^2$$
$$= \left(6 - 4\ln(3-x)\right)^2$$

2pts - integrand

1pt - answer

2. The velocity vector of a particle moving in the xy-plane has components given by

$$\frac{dx}{dt} = 14\cos\left(t^2\right)\sin\left(e^t\right) \text{ and } \frac{dy}{dt} = 1 + 2\sin\left(t^2\right), \text{ for } 0 \le t \le 1.5.$$

At time $t = 0$, the position of the particle is $(-2, 3)$.

(a) For $0 < t < 1.5$, find all values of t at which the line tangent to the path of the particle is vertical.

$\frac{dy}{dx} = \frac{1}{0}$

$\frac{dy/dt}{dx/dt} = \frac{1}{0}$ ☺

$dx/dt = 0$ $\quad dy/dt \neq 0$

$t = 1.253$, $t = 1.145$

1pt - $dx/dt = 0$

1pt - answer

(b) Write an equation for the line tangent to the path of the particle at $t = 1$.

$y - y_1 = m(x - x_1)$

$\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = .863$

$x(1) = -2 + \int_0^1 x'(t)\,dt$

$= 9.315$

$y(1) = 3 + \int_0^1 y'(t)\,dt$

$= 4.620$

$y - 4.621 = 0.863(x - 9.315)$

1pt - $\left.\frac{dy}{dx}\right|_{t=1}$

1pt - $x(1)$

1pt - $y(1)$

1pt - equation

Continue problem 2 on page 7.

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(c) Find the speed of the particle at $t = 1$.

$\hookrightarrow |v(t)|$

speed @ $t=1 = \sqrt{(x'(1))^2 + (y'(1))^2}$

$= 4.105$

1pt - answer

(d) Find the acceleration vector of the particle at $t = 1$.

$\hookrightarrow a(t) = v'(t)$
$= x''(t)$

$a(1) = \langle x''(1), y''(1) \rangle$

$= \langle -28.425, 2.161 \rangle$

1pt - $x''(1)$

1pt - $y''(1)$

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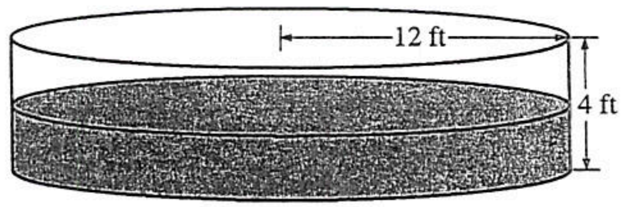
GO ON TO THE NEXT PAGE.

mpt midpt midpt

	t	0	2	4	6	8	10	12
hr	t	0	2	4	6	8	10	12
ft^3/hr	$P(t)$	0	46	53	57	60	62	63



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $\boxed{P(t)}$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $\boxed{R(t) = 25e^{-0.05t}}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

rate pumped in

rate leaked out

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

water pumped in $= \int_0^{12} P(t)\,dt = 4(46 + 57 + 62)$ — 1pt - midpt sum

$= 660 \text{ ft}^3$ — 1pt - answer

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

water leaked out $= \int_0^{12} R(t)\,dt$ — 1pt - integral

$= 225.594 \text{ ft}^3$ — 1pt - answer

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Continue problem 3 on page 9.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

↳ water pump in − water leak out

water in pool = initial amount + water pump in − water leak out

$$= 1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt$$

$$= 1434.406$$

Volume of water in pool is 1434 ft^3 @ $t = 12$ hrs

1 pt − answer

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(water in pool)' = rate pump in − rate leak out

volume water in pool → $V(t)$

$\frac{dh}{dt} = ?$ @ $t=8$

$$V'(t) = P(t) - R(t)$$

$$V'(8) = P(8) - R(8)$$

$$= 43.242 \text{ ft}^3/\text{hr}$$

1 pt − $V'(8)$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

1 pt − $\frac{dV}{dt}$ equation w/ $\frac{dh}{dt}$ in it

$$43.242 = \pi (12)^2 \frac{dh}{dt}$$

1 pt − $\left.\frac{dh}{dt}\right|_{t=8}$

$$\left.\frac{dh}{dt}\right|_{t=8} = 0.096 \text{ ft/hr}$$

1 pt − correct units for $V'(8)$ and $\frac{dh}{dt}$

r is constant

πr^2 coefficient

$V'(t) = \frac{dV}{dt}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.