



AP[®] Calculus BC
2010 Free-Response Questions
Form B

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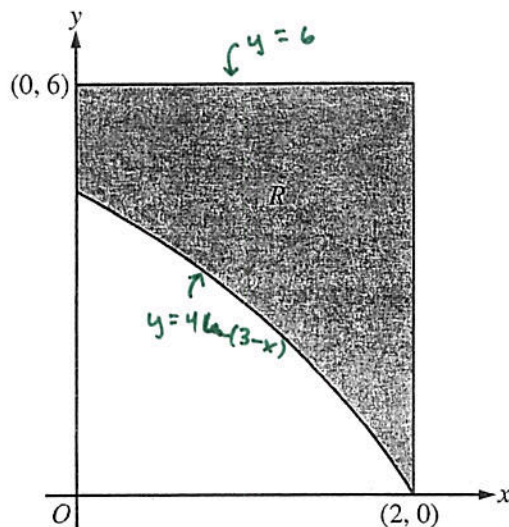
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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4 \ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R .

top-bottom .. 😊

$$\begin{aligned} \text{Area of } R &= \int_0^2 (6 - 4 \ln(3-x)) \, dx \\ &= 6.817 \end{aligned}$$

1pt - integrand

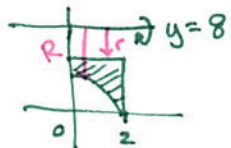
1pt - answer

1pt - correct limits in (a), (b), or (c)

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Continue problem 1 on page 5.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.



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outside radius *inside radius*

$$V = \pi \int_0^2 \left[(8 - 4 \ln(3-x))^2 - (8 - 6)^2 \right] dx$$

$$= 168.180$$

2pts - integrand
1pt - answer

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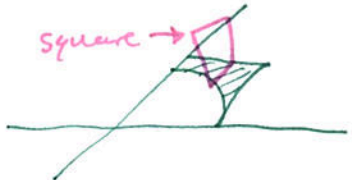
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(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of the solid.

work in x's

$$V = \int_0^2 (6 - 4 \ln(3-x))^2 dx$$

$$= 26.267$$



$$\text{Area} = (\text{side})^2$$

$$= (6 - 4 \ln(3-x))^2$$

2pts - integrand
1pt - answer

GO ON TO THE NEXT PAGE.

2. The velocity vector of a particle moving in the xy -plane has components given by

$$\frac{dx}{dt} = 14 \cos(t^2) \sin(e^t) \text{ and } \frac{dy}{dt} = 1 + 2 \sin(t^2), \text{ for } 0 \leq t \leq 1.5.$$

At time $t = 0$, the position of the particle is $(-2, 3)$.

(a) For $0 < t < 1.5$, find all values of t at which the line tangent to the path of the particle is vertical.

$\frac{dy/dt}{dx/dt} = \frac{1}{0}$
 $dx/dt = 0$ (circled)
 $t = 1.253, t = 1.145$

$\rightarrow \frac{dy}{dx} = \frac{1}{0}$

1 pt - $dx/dt = 0$

1 pt - answer

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(b) Write an equation for the line tangent to the path of the particle at $t = 1$.

$y - y_1 = m(x - x_1)$
 $\left. \frac{dy}{dx} \right|_{t=1} = \left. \frac{dy/dt}{dx/dt} \right|_{t=1} = .863$

1 pt - $\left. \frac{dy}{dx} \right|_{t=1}$

$x(1) = -2 + \int_0^1 x'(t) dt$
 $= 9.315$

1 pt - $x(1)$

$y(1) = 3 + \int_0^1 y'(t) dt$
 $= 4.621$

1 pt - $y(1)$

$y - 4.621 = 0.863(x - 9.315)$

1 pt - equation

Continue problem 2 on page 7.

2

2

2

2

2

(c) Find the speed of the particle at $t = 1$.

$$\hookrightarrow |v(t)|$$

$$\begin{aligned} \text{speed @ } t=1 &= \sqrt{(x'(1))^2 + (y'(1))^2} \\ &= 4.105 \end{aligned}$$

1 pt - answer

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(d) Find the acceleration vector of the particle at $t = 1$.

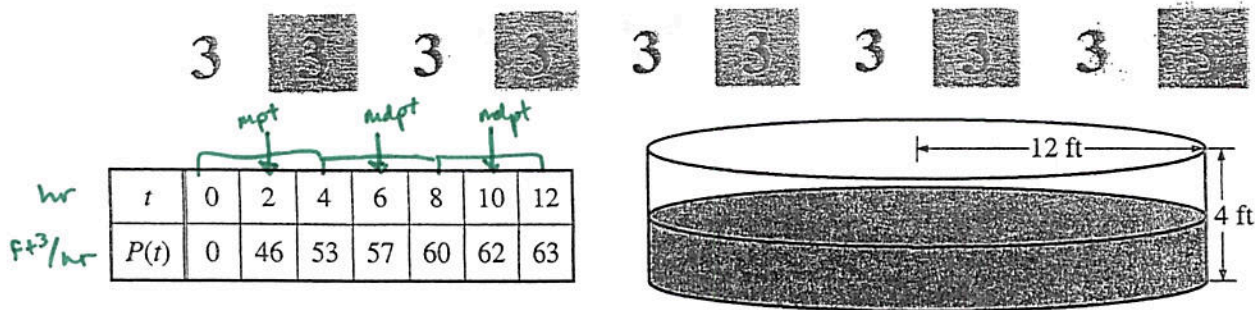
$$\begin{aligned} \hookrightarrow a(t) &= v'(t) \\ &= x''(t) \end{aligned}$$

$$\begin{aligned} a(1) &= \langle x''(1), y''(1) \rangle \\ &= \langle -28.425, 2.161 \rangle \end{aligned}$$

1 pt - $x''(1)$
1 pt - $y''(1)$

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3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t . During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

- (a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \leq t \leq 12$ hours. Show the computations that lead to your answer.

rate pumped in

water pumped in

$$\int_0^{12} P(t) dt = 4(46 + 57 + 62)$$

$$= 660 \text{ ft}^3$$

1 pt - midpoint sum

1 pt - answer

- (b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \leq t \leq 12$ hours.

water leaked out

$$= \int_0^{12} R(t) dt$$

$$= 225.594 \text{ ft}^3$$

1 pt - integral

1 pt - answer

Continue problem 3 on page 9.



- (c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

$$\text{Water in pool} = \text{initial amount} + \text{water pump in} - \text{water leak out}$$

$$= 1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt$$

$$= 1434.406$$

Volume of water in pool is 1434 ft^3
@ $t = 12 \text{ hrs}$

↪ water pump in - water leak out

1 pt - answer

- (d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

$$(\text{water in pool})' = \text{rate pump in} - \text{rate leak out}$$

$$\text{voln water in pool} \rightarrow V(t)$$

$$V'(t) = P(t) - R(t)$$

$$V'(8) = P(8) - R(8)$$

$$= 43.242 \text{ ft}^3/\text{hr}$$

$$\frac{dh}{dt} = ?$$

$$\text{@ } t = 8$$

1 pt - $V'(8)$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$43.242 = \pi (12)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} \Big|_{t=8} = 0.096 \text{ ft/hr}$$

1 pt - $\frac{dV}{dt}$ equation w/ $\frac{dh}{dt}$ in it

1 pt - $\frac{dh}{dt} \Big|_{t=8}$

1 pt - correct units for $V'(8)$ and $\frac{dh}{dt}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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