CollegeBoard
Advanced Placement Program

AP® Calculus BC
2011 Free-Response Questions

About the College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of more than 5,900 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators and schools.

Visit the College Board on the Web: www.collegeboard.org.
AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

(a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

$|v(t)|$ $\quad a(t) = v'(t)$

$$\text{speed} = \sqrt{(x'(3))^2 + (y'(3))^2}$$
$$= 13.007$$

1pt - speed

$$a(3) = \langle x''(3), y''(3) \rangle$$
$$= \langle 4, -5.467 \rangle$$

1pt - acceleration

(b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

$\left.\frac{dy}{dx}\right|_{t=3}$

$$\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right|_{t=3}$$
$$= 0.032$$

1 pt - answer

Continue problem 1 on page 5.

Do not write beyond this border.

Do not write beyond this border.

(c) Find the position of the particle at time $t = 3$.

$\hookrightarrow x(t)$
$y(t)$

$$x(3) = 0 + \int_0^3 x'(t)\,dt$$

$$= 21$$

x-coord.
1pt - integral
1pt - answer

$$y(3) = -4 + \int_0^3 y'(t)\,dt$$

$$= -3.226$$

y-coord
1pt - integral
1pt - answer

position @ $t=3$ is $(21, -3.226)$

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

$\hookrightarrow \int |v(t)|\,dt$

total distance traveled $= \displaystyle\int_0^3 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= 21.091$

1pt - integral

1pt - answer

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

$$H'(3.5) = \frac{\boxed{H(5)-H(2)}\ ^\circ C}{\boxed{5-2} \to \text{min}}$$

$$= \frac{52-60}{3}$$

$$= -8/3\ ^\circ C/\text{min}$$

1pt - answer

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

$$\frac{1}{10}\int_0^{10} H(t)\,dt = \frac{1}{10}\left[\frac{1}{2}(66+60)(2) + \frac{1}{2}(60+52)(3) + \frac{1}{2}(52+44)(4) + \frac{1}{2}(44+43)(1)\right]$$

$$\frac{1}{\text{min}}(^\circ C)(\text{min}) = 52.95\ ^\circ C$$

1pt - trap sum
1pt - estimate

$\frac{1}{10}\int_0^{10} H(t)dt$ means average temp of pot of tea in °C from $t=0$ to $t=10$ minutes.

1pt - meaning

Do not write beyond this border.

Continue problem 2 on page 7.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

$\int_0^{10} H'(t)\,dt = H(t)\Big|_0^{10}$

$= H(10) - H(0)$

$= 43 - 66$

$= -23\ ^\circ C$

1pt - value of integral

$\int_0^{10} H'(t)\,dt$ means the change in temp °C of the pot of tea from $t=0$ to $t=10$ minutes

1pt - meaning

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

$B(10) = 100 + \int_0^{10} -13.84e^{-.173t}\,dt$

$= 34.183\ ^\circ C$

1pt - integrand

1pt - initial condition

$H(10) = 43$

Biscuits are 8.817°C cooler than the tea.

1pt - answer

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Do not write beyond this border.

Do not write beyond this border.