CollegeBoard
Advanced Placement Program

# AP® Calculus BC
# 2011 Free-Response Questions
# Form B

## CALCULUS AB
## SECTION II, Part A
**Time—30 minutes**
**Number of problems—2**

**A graphing calculator is required for these problems.**

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function $S$, where $S(t)$ is measured in millimeters and $t$ is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

   (a) According to the model, what is the height of the water in the can at the end of the 60-day period?

   (b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

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Continue problem 1 on page 5.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$ ? Indicate units of measure.

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(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function $M$, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and $t$ is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function $D$ on the interval $0 \le t \le 60$ to justify that there exists a time $t$, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

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2. The polar curve $r$ is given by $r(\theta) = 3\theta + \sin\theta$, where $0 \le \theta \le 2\pi$.

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of $r$.

(b) For $\frac{\pi}{2} \le \theta \le \pi$, there is one point $P$ on the polar curve $r$ with $x$-coordinate $-3$. Find the angle $\theta$ that corresponds to point $P$. Find the $y$-coordinate of point $P$. Show the work that leads to your answers.

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Continue problem 2 on page 7.

(c) A particle is traveling along the polar curve $r$ so that its position at time $t$ is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

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**END OF PART A OF SECTION II**

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.**

**NO CALCULATOR ALLOWED**

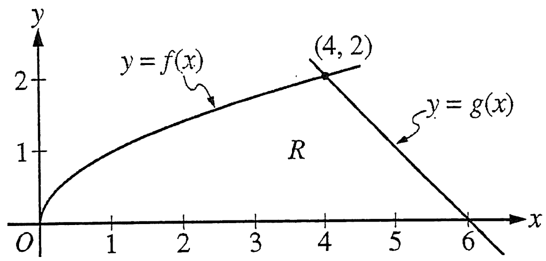
**CALCULUS AB**

**SECTION II, Part B**

**Time—60 minutes**

**Number of problems—4**

**No calculator is allowed for these problems.**

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3. The functions $f$ and $g$ are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let $R$ be the region bounded by the $x$-axis and the graphs of $f$ and $g$, as shown in the figure above.

(a) Find the area of $R$.



Continue problem 3 on page 9.

(b) The region $R$ is the base of a solid. For each $y$, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the $y$-axis is a rectangle whose base lies in $R$ and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point $P$ on the graph of $f$ at which the line tangent to the graph of $f$ is perpendicular to the graph of $g$. Find the coordinates of point $P$.

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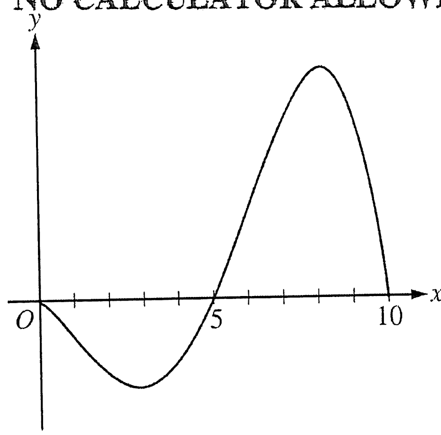
**NO CALCULATOR ALLOWED**




Graph of $f$

4. The graph of the differentiable function $y = f(x)$ with domain $0 \le x \le 10$ is shown in the figure above. The area of the region enclosed between the graph of $f$ and the $x$-axis for $0 \le x \le 5$ is 10, and the area of the region enclosed between the graph of $f$ and the $x$-axis for $5 \le x \le 10$ is 27. The arc length for the portion of the graph of $f$ between $x = 0$ and $x = 5$ is 11, and the arc length for the portion of the graph of $f$ between $x = 5$ and $x = 10$ is 18. The function $f$ has exactly two critical points that are located at $x = 3$ and $x = 8$.

(a) Find the average value of $f$ on the interval $0 \le x \le 5$.

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(b) Evaluate $\int_0^{10} (3f(x) + 2)\, dx$. Show the computations that lead to your answer.



Continue problem 4 on page 11.

(c) Let $g(x) = \int_5^x f(t)\,dt$. On what intervals, if any, is the graph of $g$ both concave up and decreasing? Explain your reasoning.

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(d) The function $h$ is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of $h$ is $h'(x) = f'\left(\frac{x}{2}\right)$. Find the arc length of the graph of $y = h(x)$ from $x = 0$ to $x = 20$.

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**NO CALCULATOR ALLOWED**

| $t$ (seconds) | 0 | 10 | 40 | 60 |
|---|---|---|---|---|
| $B(t)$ (meters) | 100 | 136 | 9 | 49 |
| $v(t)$ (meters per second) | 2.0 | 2.3 | 2.5 | 4.6 |

5. Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function $B$ models Ben's position on the track, measured in meters from the western end of the track, at time $t$, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times $t$.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

(c) For $40 \leq t \leq 60$, must there be a time $t$ when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time $t$, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$ ?

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6. Let $f(x) = \ln\left(1 + x^3\right)$.

(a) The Maclaurin series for $\ln(1 + x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1} \cdot \frac{x^n}{n} + \cdots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for $f$.

(b) The radius of convergence of the Maclaurin series for $f$ is 1. Determine the interval of convergence. Show the work that leads to your answer.

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Continue problem 6 on page 15.

(c) Write the first four nonzero terms of the Maclaurin series for $f'(t^2)$. If $g(x) = \int_0^x f'(t^2)\, dt$, use the first two nonzero terms of the Maclaurin series for $g$ to approximate $g(1)$.

(d) The Maclaurin series for $g$, evaluated at $x = 1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from $g(1)$ by less than $\frac{1}{5}$.

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