CollegeBoard
Advanced Placement
Program

AP® Calculus BC
2011 Free-Response Questions
Form B

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CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

$V = \pi r^2 h$

A graphing calculator is required for these problems.

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

$S(60) = 0 + \int_0^{60} S'(t)\,dt$

$= 171.813$ mm

1pt - limits
1pt - integrand
1pt - answer

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

avg rate of change $= \dfrac{S(60) - S(0)}{60 - 0}$ $\dfrac{\text{mm}}{\text{day}}$ ... :)

$= 2.864$ mm/day

1pt - answer

OR $\dfrac{1}{60-0}\int_0^{60} S'(t)\,dt$

$= 2.864$ mm/day

1pt - units in (b) or (c)

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Continue problem 1 on page 5.

(c) Assuming no evaporation occurs, at what $\boxed{\text{rate is the volume}}$ of water in the can changing at time $t = 7$? Indicate units of measure.

$V = \pi r^2 h$ (radius is constant) $r = 10\text{mm}$

$\frac{mm^3}{day} \rightarrow \frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$ $\quad \frac{dh}{dt} = S'(t)$

1pt - $\frac{dV}{dt}$ and $\frac{dh}{dt}$

:)

$V'(7) = \pi(10)^2 \cdot S'(7)$

$= 602.218 \ mm^3/day$

1pt - answer

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(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem (cont.) to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

$\hookrightarrow M'(t) = S'(t)$
$M'(t) - S'(t) = 0$
$D(t) = 0$

$D(0) = M'(0) - S'(0)$
$= .825 - 1.5$
$= -.675$

$D(60) = M'(60) - S'(60)$
$= 72.825 - 3.448$
$= 69.377$

1pt - $D(0)$ and $D(60)$

$D(t)$ is cont.

Since $D(0) < 0$ and $D(60) > 0$, by IVT there is some t on $(0,60)$ s.t. $D(t) = 0$

1pt - reason

$(M'(t) - S'(t) = 0 \rightarrow M'(t) = S'(t)$, so the heights are changing at same rate)

GO ON TO THE NEXT PAGE.

2. The polar curve r is given by $r(\theta) = 3\theta + \sin\theta$, where $0 \le \theta \le 2\pi$.

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of r.

$\hookrightarrow (\pi/2, \pi)$

$$\text{Area} = \frac{1}{2}\int_{\pi/2}^{\pi} (r(\theta))^2 \, d\theta$$

$$= \frac{1}{2}\int_{\pi/2}^{\pi} (3\theta + \sin\theta)^2 \, d\theta$$

$$= 47.513$$

1pt - integrand

1pt - limits + constant

1pt - answer

(b) For $\frac{\pi}{2} \le \theta \le \pi$, there is one point P on the polar curve r with x-coordinate -3. Find the angle θ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.

$\to x = r\cos\theta$

$$x = r\cos\theta = -3$$

$$(3\theta + \sin\theta)(\cos\theta) = -3$$

$$\theta = 2.017$$

$$y = r\sin\theta$$

$$y(2.017) = 6.272$$

1pt - equation $r\cos\theta = -3$

1pt - value of θ

1pt - y-coordinate

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Continue problem 2 on page 7.

(c) A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

~~$y = r\cos\theta$~~

$y = r\sin\theta$

$y = (3\theta + \sin\theta)\sin\theta$

$\frac{dy}{dt} = (\sin\theta)(3\frac{d\theta}{dt} + \cos\theta\frac{d\theta}{dt}) + (3\theta + \sin\theta)(\cos\theta\frac{d\theta}{dt})$ 1pt - use chain rule

$\left.\frac{dy}{dt}\right|_{\theta = 2\pi/3} = -2.819$ 1pt - answer

$\frac{dy}{dt} < 0 \rightarrow y$ dec

The y-coordinate is decreasing @ a rate of -2.819 1pt - meaning

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.