



## AP<sup>®</sup> Calculus BC 2012 Free-Response Questions

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$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$\begin{aligned}
 W'(12) &= \frac{W(15) - W(9)}{15 - 9} && \text{°F/min} \dots \text{☺} \\
 &= \frac{67.9 - 61.8}{6} \\
 &= 1.017 \text{ °F/min}
 \end{aligned}$$

1 pt - estimate

$W'(12)$  means that water temperature increasing at a rate of  $1.017 \text{ °F/min}$  @  $t = 12 \text{ min}$ .

OR rate of change of water temp is  $1.017 \text{ °F/min}$  @  $t = 12 \text{ min}$

1 pt - meaning w/ units

(b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.

$$\begin{aligned}
 \int_0^{20} W'(t) dt &= W(t) \Big|_0^{20} \\
 &= W(20) - W(0) && \leftarrow W \text{ measured in } \text{°F} \dots \text{☺} \\
 &= 71 - 55 \\
 &= 16 \text{ °F}
 \end{aligned}$$

1 pt - value

$\int_0^{20} W'(t) dt$  means water temperature changed  $16 \text{ °F}$  from  $t = 0$  to  $t = 20$  minutes

1 pt - meaning w/ units

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(c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\frac{1}{20} \int_0^{20} W(t) dt = \frac{1}{20} [55(4) + 57.1(5) + 61.8(6) + 67.9(5)]$$

$$= 60.79 \text{ } ^\circ\text{F}$$

$W(t)$  is increasing, so left Riemann sum is an underestimate

left-left sum approx  
left-underest w/ reason

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(d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?

*initial*

$$W(25) = 71 + \int_{20}^{25} W'(t) dt$$

$$= 73.043$$

left-integral  
left-argum

2. For  $t \geq 0$ , a particle is moving along a curve so that its position at time  $t$  is  $(x(t), y(t))$ . At time  $t = 2$ , the particle is at position  $(1, 5)$ . It is known that  $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$  and  $\frac{dy}{dt} = \sin^2 t$ .

(a) Is the horizontal movement of the particle to the left or to the right at time  $t = 2$ ? Explain your answer. Find the slope of the path of the particle at time  $t = 2$ .

$$\frac{dx}{dt} \Big|_{t=2} = \frac{2}{e^2}$$

Particle moves right @  $t=2$  b/c  $\frac{dx}{dt} \Big|_{t=2} > 0$

$$\frac{dy}{dx} \Big|_{t=2} = \frac{dy/dt}{dx/dt} \Big|_{t=2} = 3.055$$

1 pt - moves right w/ reason

1 pt -  $\frac{dy/dt}{dx/dt}$

1 pt -  $\frac{dy}{dx} \Big|_{t=2}$

(b) Find the x-coordinate of the particle's position at time  $t = 4$ .

$$x(4) = 1 + \int_2^4 x'(t) dt = 1.253$$

1 pt - integral

1 pt - answer

$(x, y)$

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(c) Find the speed of the particle at time  $t = 4$ . Find the acceleration vector of the particle at time  $t = 4$ .

$\hookrightarrow |v(t)|$

$\hookrightarrow \langle x'', y'' \rangle$

$$\text{speed @ } t=4 = \sqrt{(x'(4))^2 + (y'(4))^2}$$

$$= 0.575$$

!pt - speed

$$a(4) = \langle x''(4), y''(4) \rangle$$

$$= \langle -0.041, 0.989 \rangle$$

!pt - acceleration vector

(d) Find the distance traveled by the particle from time  $t = 2$  to  $t = 4$ .

$\hookrightarrow \int |v(t)| dt$

$$\text{Distance} = \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$= 0.651$$

!pt - integral

!pt - answer

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