AP®

AP® Calculus BC
2012 Free-Response Questions

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) = \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6} = 1.017 \text{ °F/min}$$

(°F/min) ... ☺

1pt - estimate

$W'(12)$ means that water temperature increasing at a rate of 1.017 °F/min @ $t = 12$ min.

OR rate of change of water temp is 1.017 °F/m @ $t = 12$ min

1pt - meaning w/ units

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)\,dt = W(t)\Big|_0^{20} = W(20) - W(0) = 71 - 55 = 16 \text{ °F}$$

(← W measured in °F) ... ☺

1pt - value

$\int_0^{20} W'(t)\,dt$ means water temperature changed 16°F from $t = 0$ to $t = 20$ minutes

1pt - meaning w/ units

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(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$\frac{1}{20}\int_0^{20} W(t)\,dt = \frac{1}{20}\left[55(4) + 57.1(5) + 61.8(6) + 67.9(5)\right]$

$= 60.79$ °F

1pt – left sum

1pt – approx

W(t) is increasing, so left Reimann sum is an underestimate

1pt – underest. w/ reason

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

initial

↳ $\int W'(t)$

$W(25) = 71 + \int_{20}^{25} W'(t)\,dt$ (W(20) = 71)

$= 73.043$

1pt – integral

1pt – answer

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2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

→ dy/dx $\frac{dx}{dt} < 0$ or $\frac{dx}{dt} > 0$?

$\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2}$

Particle moves right @ $t=2$ b/c $\left.\frac{dx}{dt}\right|_{t=2} > 0$

1pt – moves right w/ reason

$\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{dy/dt}{dx/dt}\right|_{t=2} = 3.055$

1pt – $\frac{dy/dt}{dx/dt}$

1pt – $\left.\frac{dy}{dx}\right|_{t=2}$

(b) Find the x-coordinate of the particle's position at time $t = 4$.

↳ $\int x'(t)dt$

$x(4) = 1 + \int_2^4 x'(t)\, dt$

$= 1.253$

1pt – integral

1pt – answer

(x, y)

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(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

↳ $|v(t)|$ ↳ $\langle x'', y'' \rangle$

Speed @ $t=4$ $= \sqrt{(x'(4))^2 + (y'(4))^2}$

$= 0.575$

1pt- speed

$a(4) = \langle x''(4), y''(4) \rangle$

$= \langle -0.041, 0.989 \rangle$

1pt - acceleration vector

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

↳ $\int |v(t)|\,dt$

Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= 0.651$

1pt- integral

1pt- ~~pos~~ answer

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