1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$\hookrightarrow G(x)$

$G'(5) = -24.588$ 1pt - $G'(5)$

$G'(t) < 0$, so $G(t)$ is dec

$G'(x)$ is neg $\Rightarrow G(x)$ is dec

The rate at which unprocessed gravel arrives at gravel processing plant @ $t = 5$ hr

- is decreasing @ 24.588 tons/hr^2

1pt - interpretation w/ units

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

total amount of unprocessed gravel $= \int_0^8 G(t)\,dt$ 1pt - integral

$= 825.551$ tons 1pt - answer

$A(t) =$ total amount of unprocessed gravel

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$A(t)$ = amount unprocessed gravel

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

$\rightarrow$ $A(t)$ inc? $A(t)$ dec?
$A'(t) > 0$? $A'(t) < 0$?

rate of unprocessed gravel = rate gravel arrives − rate gravel unprocessed

$$A'(t) = G(t) - 100$$
$$A'(5) = G(5) - 100$$
$$= -1.859$$

Amount of unprocessed gravel is decreasing @ $t=5$ b/c $A'(5)<0$

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

$0 \le t \le 8$

$\rightarrow$ abs max of $A(t)$ (use $A'(t)$ from part c) .. :)
candidates test $\rightarrow$ evaluate crit #s and endpts.

$$A'(t) = 0$$
$$G(t) - 100 = 0$$
$$G(t) = 100$$
$$t = 4.923$$

1pt - considers $A'(t)=0$

$$A(4.923) = 500 + \int_0^{4.923} (G(t) - 100)\,dt = 635.376$$

$$A(8) = 500 + \int_0^{8} (G(t) - 100)\,dt = 525.551$$

1pt - answer

$$A(0) = 500 + \int_0^{0} (G(t) - 100)\,dt = 500$$

1pt - reason

Max amount of unprocessed gravel is 635.376 tons.

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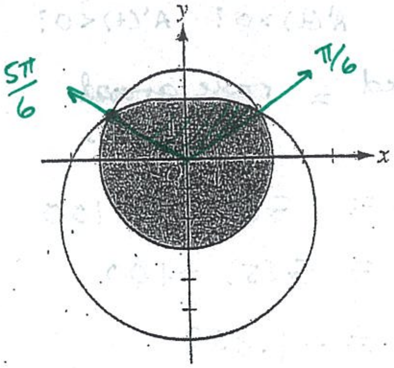

2. The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

Area of $S = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4-2\sin\theta)^2\, d\theta + \frac{2}{3}\pi(3)^2$

$= 24.709$

1pt - integrand,
1pt - limits & constant
1pt - answer

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(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\boxed{\theta = t^2}$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

$x = r\cos\theta$

$x(\theta) = (4 - 2\sin\theta)\cos\theta$ — 1pt – $x(\theta)$ or $x(t)$

$x(t) = (4 - 2\sin(t^2))\cos(t^2)$

$-1 = (4 - 2\sin(t^2))\cos(t^2)$ — 1pt – $x(\theta) = -1$ or $x(t) = -1$

$t = 1.428$ — 1pt – answer

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

position $= \langle x(t), y(t) \rangle$

$= \langle (4 - 2\sin(t^2))\cos(t^2), (4 - 2\sin(t^2))\sin(t^2) \rangle$ — 2pts – position vector

$v(1.5) = \langle x'(1.5), y'(1.5) \rangle$

$= \langle -8.072, -1.673 \rangle$ — 1pt – velocity vector

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