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AB

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

$$\frac{1}{6}\int_0^6 C(t)\,dt = \frac{1}{6}\left[2(5.3) + 2(11.2) + 2(13.8)\right]$$

$$= \frac{1}{6}\left[2(5.3 + 11.2 + 13.8)\right]$$

$$= \frac{1}{3}(30.3)$$

$$= 10.1 \text{ ounces}$$

1pt – mpt sum
1pt – approximation

$\frac{1}{6}\int_0^6 C(t)\,dt$ means the average # of ounces in the coffee cup from $t=0$ to $t=6$ minutes

1pt – meaning

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

$B'(t)$

$$B'(t) = -16e^{-.4t} \cdot -.4$$

1pt – $B'(t)$

$$B'(5) = -16e^{-.4(5)}(-.4)$$

$$= 6.4e^{-2} \text{ ounces/min}$$

1pt – $B'(5)$

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t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$C'(3.5) = \frac{C(4) - C(3)}{4 - 3}$$

$$= \frac{12.8 - 11.2}{1}$$

$$= 1.6 \text{ ounces/min}$$

1pt – approximation

1pt – units

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

MVT?

C(t) cont? yes, b/c C(t) diff'able
C(t) diff'able? yes, given

$$\frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{4 - 2}$$

$$= \frac{4}{2}$$

$$= 2$$

1pt – $\frac{C(4)-C(2)}{4-2}$

Since C(t) is cont + diff'able, there is a time t, on [2,4] at which $C'(t) = 2$.

1pt – conclusion w/ MVT

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Continue problem 3 on page 15.

AB

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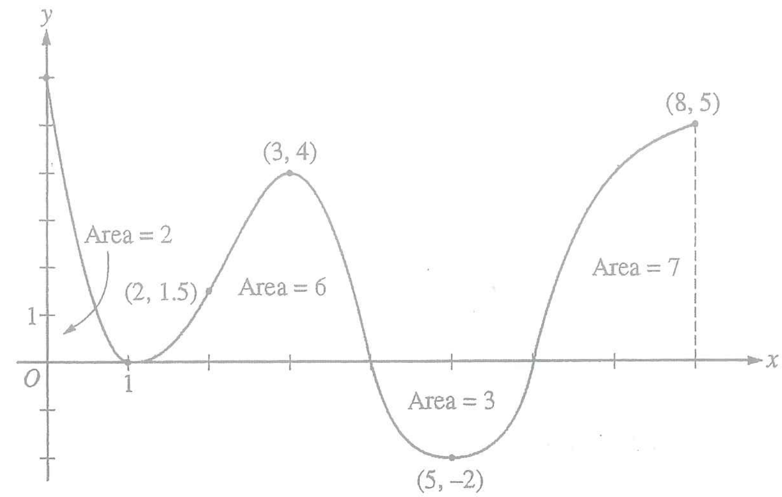

Graph of f'

f': + + − + (1, 4, 6)

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

↳ f' from neg to pos

f has rel. min @ $x = 6$ b/c

f' changes from neg to pos @ $x = 6$

1pt - answer w/ reason

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

→ rel. min or endpts.

$f(0) = 4 + \int_8^0 f'(x)\,dx = 4 - \int_0^8 f'(x)\,dx$

$= 4 - (2 + 6 - 3 + 7)$

$= -8$

$f(6) = 4 + \int_8^6 f'(x)\,dx = 4 - \int_6^8 f'(x)\,dx$

$= 4 - (7) = -3$

$f(8) = 4$

abs min value is -8

1pt - considers $x=0$ and $x=6$

1pt - answer

1pt - reason

Continue problem 4 on page 17.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

$\hookrightarrow f'' < 0$ $\hookrightarrow f' > 0$

f' dec

f inc → $f' > 0$ on $(0,4) \cup (6,8)$

f conc. down → f' dec on $(0,1) \cup (3,5)$

1pt - answer
1pt - reason

f concave down + inc on $(0,1) \cup (3,4)$

b/c f' dec and f' pos. on $(0,1) \cup (3,4)$

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

$g'(x)$

$g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(3) = 3(f(3))^2 \cdot f'(3)$

$= 3(-\frac{5}{2})^2 \cdot 4$

$= 3 \cdot \frac{25}{4} \cdot 4$

$= 75$

2pts - $g'(x)$

1pt - answer

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5. Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\lim_{x\to 0} \frac{f(x)+1}{\sin x}$. Show the work that leads to your answer.

$$\frac{\lim_{x\to 0} f(x)+1}{\lim_{x\to 0} \sin x} = \frac{-1+1}{\sin 0} = \frac{0}{0} \quad \text{L'Hôpital}$$

1pt – L'Hôpital

$$\lim_{x\to 0} \frac{f'(x)}{\cos x} = \frac{f'(0)}{\cos 0} = \frac{(-1)^2(2\cdot 0+2)}{1}$$

$$= 2$$

1pt – answer

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

$\Delta x = \frac{1/2 - 0}{2} = \frac{1}{4}$

	Δx	$\frac{dy}{dx}$	$\Delta x \frac{dy}{dx}$	$y + \Delta x \frac{dy}{dx}$
$(0,-1)$	$1/4$	2	$1/2$	$-1 + 1/2 = -1/2$
$(1/4, -1/2)$	$1/4$	$5/8$	$5/32$	$-1/2 + 5/32 = -\frac{11}{32}$
$(1/2,$				

$(-\frac{1}{2})^2(2\cdot\frac{1}{4}+2)$

$\frac{1}{4}(\frac{1}{2}+2)$

$\frac{1}{4}(\frac{5}{2})$

$\frac{5}{8}$

$\frac{-16}{32} + \frac{5}{32}$

$\frac{-11}{32}$

$f(\frac{1}{2}) = -\frac{11}{32}$

1 pt – Euler w/2 steps

1pt – answer

Continue problem 5 on page 19.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

↳ x dx's, y, dy's

$$\frac{dy}{dx} = y^2(2x+2)$$

$$\int \frac{1}{y^2}\,dy = \int (2x+2)\,dx$$ 1pt - separate variable

$$\int y^{-2}\,dy = x^2 + 2x + C$$ 1pt - antiderive

1pt - "+C"

$$-\frac{1}{y} = x^2 + 2x + C$$

$$-\frac{1}{-1} = 0^2 + 2(0) + C$$ 1pt - initial condition

$$1 = C$$

$$-\frac{1}{y} = x^2 + 2x + 1$$

$$-\frac{1}{y} = \frac{x^2+2x+1}{1}$$

$$-y = \frac{1}{x^2+2x+1}$$

$$y = -\frac{1}{x^2+2x+1}$$ 1pt - solve for y

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6. A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about $x = 0$.

(a) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.

$P_1(x) = f(a) + f'(a)(x-a)$

$P_1(x) = f(0) + f'(0)(x-0)$ 1pt - $P'(x)$

$P_1(x) = -4 + f'(0)(x)$

$P_1(\frac{1}{2}) = -4 + f'(0) \cdot \frac{1}{2}$

$-3 = -4 + \frac{1}{2} f'(0)$

$1 = \frac{1}{2} f'(0)$

$2 = f'(0)$ 1pt - shows $f'(0) = 2$

(b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.

$P_3(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$

$= f(0) + f'(0)(x) + \frac{f''(0)}{2} \cdot x^2 + \frac{f'''(0)}{3!} x^3$

$= -4 + 2x + \frac{-2/3}{2} x^2 + \frac{1/3}{3!} x^3$

1pt - 1st 2 terms
1pt - 3rd term
1pt - 4th term

OR

$= -4 + 2x - \frac{1}{3} x^2 + \frac{1}{18} x^3$

Continue problem 6 on page 21.

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(c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.

$$T_3(x) = h(a) + h'(a)(x-a) + \frac{h''(a)}{2!}(x-a)^2 + \frac{h'''(a)}{3!}(x-a)^3$$

$h(0) = 7$

$h'(x) = f(2x)$

$h'(0) = f(2\cdot 0)$

$= f(0)$

$= -4$

$h''(x) = f'(2x)\cdot 2$

$h''(0) = f'(0)\cdot 2$

$= 2\cdot 2$

$= 4$

$h'''(x) = 2f''(2x)\cdot 2$

$h'''(0) = 2f''(0)\cdot 2$

$= 4(-2/3)$

$= -8/3$

2pts - ~~uses~~ applies $h'(x) = f(2x)$

1pt - constant term

1pt - remaining terms

$$T_3(x) = h(0) + h'(0)(x) + \frac{h''(0)}{2!}(x)^2 + \frac{h'''(0)}{3!}\cdot x^3$$

$$= 7 - 4x + \frac{4}{2!}x^2 + \frac{-8/3}{3!}x^3$$

or

$$7 - 4x + 2x^2 - \frac{4}{9}x^3$$

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