AP®

AP® Calculus BC
2014 Free-Response Questions

CollegeBoard

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

avg rate of change of A(t) $= \dfrac{A(30) - A(0)}{30 - 0}$

$\underline{A(t) \to \text{lbs}}$
$t \to \text{days}$ ... ☺

$= -.197$ lbs/day

1pt - answer w/units

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$A'(15) = -.164$ lbs/day

1pt - $A'(15)$

☺

$f'(x) < 0$ means $f(x)$ dec

$\therefore A'(15) < 0$ means $A(15)$ dec

$A'(15)$ is amount of grass clippings @ $t = 15$ days is decreasing at rate of .164 lbs/day.

1pt - explanation

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Continue problem 1 on page 5.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

$A(t)$ $=$

$\frac{1}{b-a}\int_a^b A(t)\,dt$

$$A(t) = \frac{1}{30-0}\int_0^{30} A(t)\,dt$$

1pt - $\frac{1}{30}\int_0^{30} A'(t)dt$

$$6.687(0.931)^t = \frac{1}{30}\int_0^{30} A(t)\,dt$$

$$t = 12.415$$

1pt - answer

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$$y - y_1 = m(x - x_1)$$

$$\left.\begin{aligned} &L(t) - A(30) = A'(30)(t-30) \\ &L(t) = A'(30)(t-30) + A(30) \end{aligned}\right\} \text{2pts - } L(t)$$

$$0.5 = A'(30)(t-30) + A(30)$$

1pt - sub $L(t) = 0.5$

$$t = 35.054$$

1pt - answer

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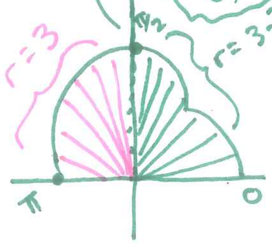
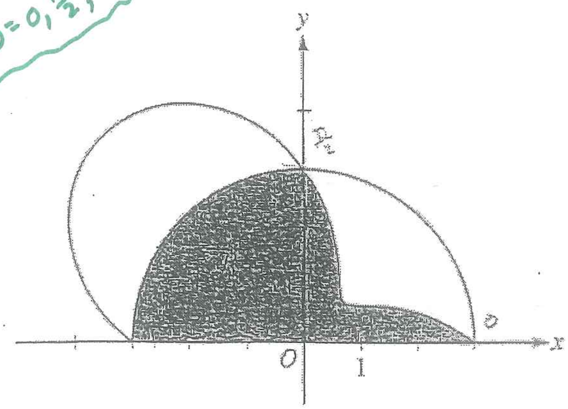
$3-2\sin 2\theta = 3$

$0 = 2\sin 2\theta$

$0 = \sin 2\theta$

$0, \pi, 2\pi = 2\theta \rightarrow \theta = 0, \frac{\pi}{2}, \pi$





2. The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.

   (a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

$$\text{Area of } R = \int_0^{\pi/2} \frac{1}{2}(3 - 2\sin 2\theta)^2\, d\theta + \int_{\pi/2}^{\pi} \frac{1}{2}(3)^2\, d\theta$$

$$= 9.708$$

1pt - integrand
1pt - limits
1pt - answer

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Continue problem 2 on page 7.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

$x = r\cos\theta$

$= (3 - 2\sin 2\theta)(\cos\theta)$

$\left.\frac{dx}{d\theta}\right|_{\theta = \pi/6} = -2.366$

1pt - expression for x

1pt - answer

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{((3-2\sin 2\theta)(\cos\theta) - 3\cos\theta)^2 + ((3-2\sin 2\theta)(\sin\theta) - 3\sin\theta)^2}$

$\left.\frac{dd}{d\theta}\right|_{\theta = \pi/3} = -2$

or distance $= r_1 - r_2$

$= 3 - (3 - 2\sin 2\theta)$

$= 2\sin(2\theta)$

$\left.\frac{dd}{d\theta}\right|_{\theta = \pi/3} = -2$

1pt - expression for distance

1pt - answer

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.

$\cancel{\frac{dr}{d\theta}}$ $\frac{dr}{dt} = -2\cos(2\theta) \cdot 2\frac{d\theta}{dt}$

$= -2\cos\left(2 \cdot \frac{\pi}{6}\right) \cdot 2(3)$

$\frac{dr}{dt} = -6$

1pt - chain rule w/ respect to t

1pt - answer

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