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# AP<sup>®</sup> Calculus BC

## 2015 Free-Response Questions

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1. The rate at which rainwater flows into a drainpipe is modeled by the function  $R$ , where  $R(t) = 20 \sin\left(\frac{t^2}{35}\right)$  cubic feet per hour,  $t$  is measured in hours, and  $0 \leq t \leq 8$ . The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by  $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$  cubic feet per hour, for  $0 \leq t \leq 8$ . There are 30 cubic feet of water in the pipe at time  $t = 0$ .
- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval  $0 \leq t \leq 8$ ?

- (b) Is the amount of water in the pipe increasing or decreasing at time  $t = 3$  hours? Give a reason for your answer.

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Continue problem 1 on page 5.

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(c) At what time  $t$ ,  $0 \leq t \leq 8$ , is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For  $t > 8$ , water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time  $w$  when the pipe will begin to overflow.

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2. At time  $t \geq 0$ , a particle moving along a curve in the  $xy$ -plane has position  $(x(t), y(t))$  with velocity vector  $v(t) = (\cos(t^2), e^{0.5t})$ . At  $t = 1$ , the particle is at the point  $(3, 5)$ .

(a) Find the  $x$ -coordinate of the position of the particle at time  $t = 2$ .



(b) For  $0 < t < 1$ , there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

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Continue problem 2 on page 7.

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(c) Find the time at which the speed of the particle is 3

(d) Find the total distance traveled by the particle from time  $t = 0$  to time  $t = 1$ .

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$t$ (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

3. Johanna jogs along a straight path. For  $0 \leq t \leq 40$ , Johanna's velocity is given by a differentiable function  $v$ . Selected values of  $v(t)$ , where  $t$  is measured in minutes and  $v(t)$  is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of  $v'(16)$ .

(b) Using correct units, explain the meaning of the definite integral  $\int_0^{40} |v(t)| dt$  in the context of the problem.

Approximate the value of  $\int_0^{40} |v(t)| dt$  using a right Riemann sum with the four subintervals indicated in the table.

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- (c) Bob is riding his bicycle along the same path. For  $0 \leq t \leq 10$ , Bob's velocity is modeled by  $B(t) = t^3 - 6t^2 + 300$ , where  $t$  is measured in minutes and  $B(t)$  is measured in meters per minute.

Find Bob's acceleration at time  $t = 5$ .

- (d) Based on the model  $B$  from part (c), find Bob's average velocity during the interval  $0 \leq t \leq 10$ .

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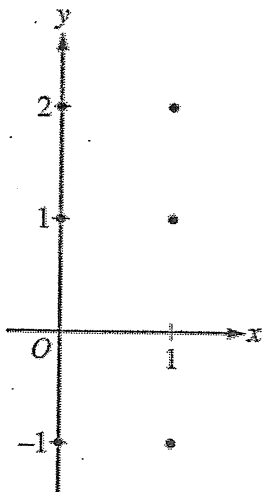
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4. Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

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- (c) Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition:  $f(2) = 3$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 2$ ? Justify your answer.



- (d) Find the values of the constants  $m$  and  $b$  for which  $y = mx + b$  is a solution to the differential equation.

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5. Consider the function  $f(x) = \frac{1}{x^2 - kx}$ , where  $k$  is a nonzero constant. The derivative of  $f$  is given by

$$f'(x) = \frac{k - 2x}{(x^2 - kx)^2}$$

(a) Let  $k = 3$ , so that  $f(x) = \frac{1}{x^2 - 3x}$ . Write an equation for the line tangent to the graph of  $f$  at the point whose  $x$ -coordinate is 4.

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(b) Let  $k = 4$ , so that  $f(x) = \frac{1}{x^2 - 4x}$ . Determine whether  $f$  has a relative minimum, a relative maximum, or neither at  $x = 2$ . Justify your answer.

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(c) Find the value of  $k$  for which  $f$  has a critical point at  $x = -5$ .



(d) Let  $k = 6$ , so that  $f(x) = \frac{1}{x^2 - 6x}$ . Find the partial fraction decomposition for the function  $f$ .

Find  $\int f(x) dx$ .

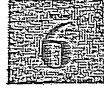


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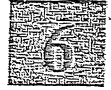
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6. The Maclaurin series for a function  $f$  is given by  $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \dots + \frac{(-3)^{n-1}}{n} x^n + \dots$  and converges to  $f(x)$  for  $|x| < R$ , where  $R$  is the radius of convergence of the Maclaurin series.

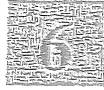
(a) Use the ratio test to find  $R$ .

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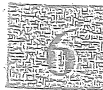
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- (b) Write the first four nonzero terms of the Maclaurin series for  $f'$ , the derivative of  $f$ . Express  $f'$  as a rational function for  $|x| < R$ .

- (c) Write the first four nonzero terms of the Maclaurin series for  $e^x$ . Use the Maclaurin series for  $e^x$  to write the third-degree Taylor polynomial for  $g(x) = e^x f(x)$  about  $x = 0$ .

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