AP

AP® Calculus BC
2015 Free-Response Questions

CollegeBoard

rate/derivative in

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are $\boxed{\text{30 cubic feet}}$ of water in the pipe at time $t = 0$.

initial

rate out

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

↳ $\int$ rate

rainwater flows into pipe $= \int_0^8 R(t)\,dt$

$= 76.570$ ft^3

1pt: integrand

1pt: answer

(units not required)

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

rate > 0 water in pipe or rate < 0 water in pipe @ t = 3

rate water in pipe = rate flow in − rate drain out

$= R(3) - D(3)$

$= -.314$

1pt: considers R(3) & D(3)

Amount of water in pipe is decreasing @ t = 3

b/c $R(3) - D(3) < 0$

1pt: answer w/reason

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Continue problem 1 on page 5.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

rate water in pipe = 0

↳ rel. min → f' changes neg to pos, + check endpts.

$R(t) - D(t) = 0$

$R(t) = D(t)$

$t = 0,\ t = 3.272$

1pt : consider $R(t) - D(t) = 0$

water in pipe @ $t = 0 \rightarrow 30 + \int_0^0 (R(t) - D(t))\,dt = 30$

@ $t = 3.272 \rightarrow 30 + \int_0^{3.272} (R(t) - D(t))\,dt = 27.965$

@ $t = 8 \rightarrow 30 + \int_0^{8} (R(t) - D(t))\,dt = 48.544$

1pt : answer

1pt : justification

Amount of water in pipe is a minimum @ $t = 3.272$

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

$$30 + \int_0^{w} (R(t) - D(t))\,dt = 50$$

1pt : integral

1pt : equation

GO ON TO THE NEXT PAGE.

2. At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

$\hookrightarrow \int v(t)$

$$x(2) = x(1) + \int_1^2 \cos(t^2)\,dt$$

$$= 3 + \int_1^2 \cos(t^2)\,dt$$

$$= 2.557$$

1pt: integral

1pt: initial condition

1pt: answer

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

$\hookrightarrow dy/dx = 2$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$2 = \frac{e^{0.5t}}{\cos(t^2)}$$

$$t = 0.840$$

1pt: slope in terms of t

1pt: answer

Continue problem 2 on page 7.

(c) Find the time at which the speed of the particle is 3.

$\hookrightarrow |v(t)| = \sqrt{(x')^2 + (y')^2}$

$$\text{Speed} = \sqrt{(\cos t^2)^2 + (e^{0.5t})^2}$$

$$3 = \sqrt{(\cos t^2)^2 + (e^{0.5t})^2}$$

$$t = 2.196$$

1pt: speed in terms of t

1pt: answer

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

$\hookrightarrow \int |v(t)|$

$$\text{Total distance} = \int_0^1 \sqrt{(\cos t^2)^2 + (e^{.5t})^2}\, dt$$

$$= 1.595$$

1pt: integral

1pt: answer

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