1 2 3 2

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

$$R'(2) = \frac{R(3) - R(1)}{3 - 1}$$

$\frac{\text{Liters/hr}}{\text{hr}}$ ...

$$= \frac{950 - 1190}{3 - 1}$$

$$= -120 \text{ Liters/hr}^2$$

1pt: estimate

1pt: units

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

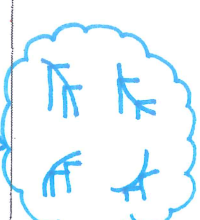
$$\int_0^8 R(t)\,dt \approx 1(1340) + 2(1190) + 3(950) + 2(740)$$

$$= 8050 \text{ Liters}$$

1pt: left Riemann sum

1pt: estimate

Left rectangles



R is decreasing on $(0,8)$,

$\therefore$, the estimate is an overestimate

1pt: over w/ reason

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Continue problem 1 on page 5.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

total amount water in tank = initial amount + water in − water out

$= 50000 + \int_0^8 W(t)\,dt - \int_0^8 R(t)\,dt$

1pt: integral

$= 50000 + \int_0^8 W(t)\,dt - 8050$

$= 49786.195$

$= 49786$ liters

1pt: estimate

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

will Rate pumped in = Rate removed ?

$W(t) = R(t)$?

$W(t) - R(t) = 0$?

$W(0) - R(0) = 48000$

$W(8) - R(8) = -618.476$

1pt: considers $W(t) - R(t)$

Yes, there is a time on $(0,8)$, by IVT,

since $W(0) - R(0) < 0$ and $W(8) - R(8) > 0$,

$\therefore W(t) - R(t) = 0$ on $(0,1)$

1pt: answer w/ explanation

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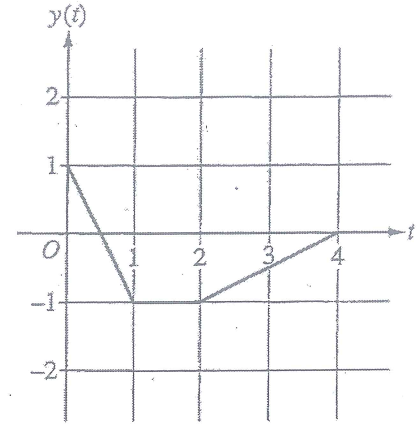

2. At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

$$\text{position} = \langle x(3),\ y(3) \rangle$$

$$= \langle x(0) + \int_0^3 x'(t)\,dt,\ y(3) \rangle$$

$$= \langle 5 + \int_0^3 x'(t)\,dt,\ y(3) \rangle$$

$$= \langle 14.377,\ -\tfrac{1}{2} \rangle$$

1pt: integral
1pt: use initial condition
1pt: answer

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

$$\text{slope of tangent line} = \frac{dy}{dx}$$

$$\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right|_{t=3}$$

$$= \frac{1/2}{3^2 + \sin(3 \cdot 3^2)}$$

$$= 0.050$$

1pt: slope

Continue problem 2 on page 7.

(c) Find the speed of the particle at $t = 3$.

$$\text{speed} = \sqrt{(x'(t))^2 + (y'(t))^2}$$

$$\underset{@\ t=3}{\text{speed}} = \sqrt{(x'(3))^2 + (y'(3))^2}$$

$$= 9.969$$

1pt: expression for speed

1pt: answer

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(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$\hookrightarrow \int |v(t)|$

$$\text{total distance} = \int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

1pt: expression for distance

$$= \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt + \int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$$= \int_0^1 \sqrt{(x'(t))^2 + (-2)^2}\, dt + \int_1^2 \sqrt{(x'(t))^2 + 0^2}\, dt$$

1pt: integrals

$$= 4.350$$

1pt: answer

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