$\Delta h = 2 \quad \Delta h = 3 \quad \Delta h = 5$

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

$(ft^2)(ft) = ft^3$

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

$\int^{10} A(h)\,dh = 2(50.3) + 3(14.4) + 5(6.5) \quad ft^3$

← ok to stop here

Volume of the tank $= 176.3 \ ft^3$

1pt - left sum
1pt - approx

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

The approx in part (a) is an overestimate of volume of tank b/c A is decreasing

1pt - overestimate w/ reason

Continue question 1 on page 5.

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $\boxed{f(h) = \dfrac{50.3}{e^{0.2h} + h}}$. Based on this model, find the volume of the tank. Indicate units of measure.

Area of cross section $= f(h)$

$$\text{Volume} = \int_0^{10} \frac{50.3}{e^{.2h} + h}\, dh$$

$= 10.325 \text{ ft}^3$

1pt - integral

1pt - answer

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

$h = 5\text{ft}$

$\frac{dh}{dt} = .26\text{ft/min}$

use $\int f(h) = $ volume

$\frac{dV}{dt} = ?$

$$\text{Volume} = \int_0^h f(H)\, dH$$

$$\frac{dV}{dt} = f(h) \cdot \frac{dh}{dt}$$

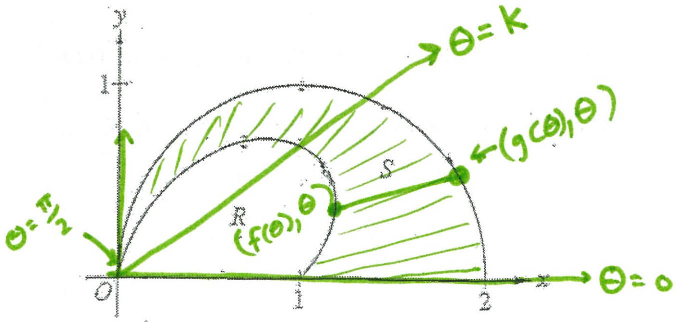
$= f(5) \cdot (.26)$

$= 1.694 \text{ ft}^3/\text{min}$

2pts → $\frac{dV}{dt}$

1pt → answer

1pt → correct units in (a), (c) and (d)



2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $r = g(\theta) = 2\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x-axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x-axis.

(a) Find the area of R.

Area of $R = \frac{1}{2}\int_0^{\pi/2} (f(\theta))^2\, d\theta$

$= 0.648$

1pt → integral
1pt → answer

(b) The ray $\boxed{\theta = k}$ where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

Area of $S = \frac{1}{2}\int_0^{\pi/2} (g(\theta))^2\, d\theta - \frac{1}{2}\int_0^{\pi/2} (f(\theta))^2\, d\theta$

$\frac{1}{2}\int_0^{k} \left[(g(\theta))^2 - (f(\theta))^2\right] d\theta = \frac{1}{2}\int_k^{\pi/2} \left[(g(\theta))^2 - (f(\theta))^2\right] d\theta$

"OR"

$\dfrac{\frac{1}{2}\int_0^{\pi/2} \left[(g(\theta))^2 - (f(\theta))^2\right] d\theta}{2} = \frac{1}{2}\int_0^{k} \left[(g(\theta))^2 - (f(\theta))^2\right] d\theta$

1pt → integral for one area region
1pt → equation

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Continue question 2 on page 7.

(c) For each θ, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

$\frac{1}{b-a}\int_a^b$

$w(\theta) = g(\theta) - f(\theta)$

1pt → $w(\theta)$

$$w_A = \frac{1}{\pi/2 - 0}\int_0^{\pi/2} (g(\theta) - f(\theta))\, d\theta$$

1pt → integral

$= 0.485$

1pt → answer

(d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

$w' > 0$ or < 0?

$w(\theta) = w_A$

$g(\theta) - f(\theta) = 0.485$

$\theta = 0.518$

1pt → solves $w(\theta) = w_A$

$w'(\theta) = g'(\theta) - f'(\theta)$

$w'(0.518) = -.582$

1pt → answer w/ reason

$w(\theta)$ is dec @ $\theta = 0.518$ b/c $w'(0.518) < 0$

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