2018

AP® CollegeBoard

AP Calculus BC

Free-Response Questions

1. People enter a line for an escalator at a rate modeled by the function r given by

derivative
$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are $\boxed{\text{20 people in line}}$ at time $t = 0$.

derivative

initial

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

↳ ∫ rate enter

$$\#\text{people enter line} = \int_0^{300} r(t)\,dt$$

1pt - integral

1pt - answer (units not required)

$$= 270 \text{ people}$$

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

↳ # in line + #arrive − #leave
+ ∫ rate − ∫ rate

$$\text{people in line @ } t=300 = 20 + \int_0^{300} (r(t) - 0.7)\,dt$$

1pt - considers rate out

1pt - answer

$$= 80$$

Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

start @ t=300

→ 0 = # in line + # arrive − # leave

$$0 = 20 + \int_{300}^{t} (r(x) - 0.7)\,dx$$

$r(x) = 0$ for $t > 300$

$$0 = 20 + \int_{300}^{t} (0 - 0.7)\,dx$$

$$0 = 20 + -0.7x\Big|_{300}^{t}$$

$$0 = 20 + -0.7t - (-0.7(300))$$

$$t = 414.286 \text{ seconds}$$

1pt - answer

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

t = ? abs min P = ?

→ crit #s + endpts into original eq.

$$p(t) = \#\text{ people in line} = 20 + \int_0^t (r(x) - .7)\,dx$$

$$p'(t) = r(t) - .7$$

$$0 = r(t) - .7$$

$$r(t) = .7$$

$$t = 33.013$$

1pt: considers $r(t) - 0.7 = 0$

1pt: $t = 33.013$

$$P(33.013) = 20 + \int_0^{33.013} (r(t) - .7)\,dt$$

$$= 3.803$$

$$P(0) = 20$$

$$P(300) = 80$$

1pt: justification

1pt: answers

@ t = 33.013 sec, # of people in line is a minimum.

# of people in line @ t = 33.013 is 4 people

units

units

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of $\boxed{h \text{ meters}}$, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'>0 \rightarrow p$ inc
$p'<0 \rightarrow p$ dec

$p'(25) = -1.179$

The density of plankton cells is decreasing @ rate of 1.179 million cells/meter^3 @ h=25 meters

1pt: answer
1pt: meaning w/units

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$$\#\text{ cells btn } h=0 \text{ \& } h=30 = \int_0^{30} 3 \cdot p(h)\, dh$$

$= 1675.415$

1675 million plankton cells

1pt: integrand
1pt: answer

Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\boxed{\int_{30}^{\infty} u(h)\,dh = 105}$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

h ≥

use f(h) ... h≥30

$$\int_{30}^{K} 3f(h)\,dh + \int_{0}^{30} 3p(h)\,dh = \text{plankton in entire column}$$

1pt: integral expression

$$= \int_{30}^{K} 3f(h)\,dh + 1675.415$$

since $0 \le f(h) \le u(h)$.. $\int_{30}^{K} 3f(h)\,dh \le \int_{30}^{\infty} 3\,u(h)\,dh$

1pt: compares improper integral

$$3\int_{30}^{k} f(h)\,dh + 1675.415 \le 3\int_{30}^{\infty} u(h)\,dh + 1675.415$$

$$\le 3(105) + 1675.415$$

$$\le 1990.415 \le 2000$$

million cells

1pt: answer

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$\int |v(t)|\,dt$

$$\text{total distance} = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

1pt: integrand

$$= 757.456 \text{ meters}$$

1pt: total distance answer

GO ON TO THE NEXT PAGE.