2021

AP CollegeBoard

AP® Calculus BC

Free-Response Questions

Answer QUESTION 1 parts (a) and (b) on this page.

Δr = 1, 1, 0.5, 1.5

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2}$$

1 pt: estimate

$$\approx \frac{10 - 6}{2.5 - 2}$$

← ok to stop here

$$= 8 \text{ mg/cm}^3$$

← units not required here

$f'(2.25)$ means density of bacteria population in circular petri dish is inc @ 8 mg/cm³ when $r = 2.25$ cm

1 pt: interpretation w/ units

Response for question 1(b)

$$2\pi \int_0^4 r f(r)\, dr \approx 2\pi \left[4(18)(1.5) + 2.5(10)(0.5) + 2(6)(1) + 1(2)(1) \right]$$

($r \cdot f(r)$ function, Δr)

1 pt: right Riemann sum

1 pt: approx

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

rf(r) is inc? ☺

1pt: product rule for rf(r)

$h(r) = rf(r)$

$h'(r) = f(r) + rf'(r)$ (>0?)

$f(r) > 0$ on $[0,4]$ and $rf'(r) > 0$ on $[0,4]$, so $h'(r) > 0$

$\therefore$, $h(r)$ is inc.

The approx in part b is overestimate of total mass b/c $rf(r)$ is inc.

1pt: answer w/ explanation

Response for question 1(d)

$g(k) = \frac{1}{b-a}\int_a^b g(r)\,dr$ ☺

1pt: definite integral

1pt: average value

$2 - 16(\cos 1.57\sqrt{k})^3 = \frac{1}{4-1}\int_1^4 g(r)\,dr$

1pt: answer

$k = 7.497$

☺ nsolve

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

speed $= |v(1.2)|$

$= \sqrt{((1.2-1)e^{(1.2)^2})^2 + (\sin(1.2^{1.25}))^2}$

$= 1.271$

1 pt: speed

$a(1.2) = v'(1.2)$

$= \langle x''(1.2), y''(1.2) \rangle$

$= \langle 6.247, 0.405 \rangle$

1 pt: acceleration ~~velocity~~ vector

Response for question 2(b)

$v(t) = \langle x'(t), y'(t) \rangle$

total distance $= \int_0^1 |v(t)|\, dt$

$= \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

$= 1.010$

1 pt: integrand

1 pt: answer

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

Farthest left → abs min (x(t) only) ☺

$x'(t) = (t-1)e^{t^2} = 0$

$t = 1$

1pt: $x'(t) = 0$

1pt: leftmost point @ $t = 1$

1pt: one coordinate of leftmost point

$x(1) = x(0) + \int_0^1 x'(t)\,dt$

$= -2 + \int_0^1 x'(t)\,dt$

$= -2.604$

Particle is furthest left @ $t = 1$

b/c $x'(t) < 0$ on $(0,1)$ and $x'(t) > 0$ on $(1,\infty)$ ☺

explain this, so no need to check endpts

$y(1) = y(1) + \int_0^1 y'(t)\,dt$

$= 5 + \int_0^1 y'(t)\,dt$

$= 5.410$

Particle is farthest left @ $\langle -2.604, 5.410 \rangle$

1pt: Left most point

$x'(t) > 0$ on $(1,\infty)$, ∴, particle moves right on $(1,\infty)$

$\int_0^\infty x'(t)\,dt = \infty$,

∴ particle does not have a pt. farthest to the right

1pt: explanation