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# AP<sup>®</sup> Calculus BC

## Free-Response Questions

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Answer QUESTION 1 parts (a) and (b) on this page.

$r$ (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

*Handwritten notes above table:*  $\Delta r = 1$  (between 0 and 1),  $0.5$  (between 2 and 2.5),  $1.5$  (between 2.5 and 4). Below the table, there are red 'R' marks under the columns for  $r=1, 2, 2.5, 4$ .

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2}$$

*1pt: estimate*

$$\approx \frac{10 - 6}{2.5 - 2}$$

*ok to stop here*

$$= 8 \text{ mg/cm}^3$$

*units not required here*

$f'(2.25)$  means density of bacteria population in circular petri dish is inc @  $8 \text{ mg/cm}^3$  when  $r = 2.25 \text{ cm}$

*1pt: interpretation w/ units*

Response for question 1(b)

$$2\pi \int_0^4 r f(r) dr \approx 2\pi \left[ \underbrace{4(18)}_{\substack{r \cdot f(r) \\ \text{function}}} (1.5) + \underbrace{2.5(10)}_{\substack{r \cdot f(r) \\ \text{function}}} (0.5) + \underbrace{2(6)}_{\substack{r \cdot f(r) \\ \text{function}}} (1) + \underbrace{1(2)}_{\substack{r \cdot f(r) \\ \text{function}}} (1) \right]$$

*Handwritten notes above the integral:*  $\Delta r$  (between 0 and 1),  $0.5$  (between 2 and 2.5),  $1$  (between 2.5 and 4),  $1$  (between 4 and 4). Arrows point from the  $\Delta r$  labels to the corresponding terms in the sum.

*1pt: right Riemann sum*  
*1pt: approx*

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

$rf(r)$  is inc? ☺  
 $h(r) = rf(r)$   
 $h'(r) = f(r) + rf'(r) > 0?$

1pt: product rule for  $rf(r)$

$f(r) > 0$  on  $[0, 4]$  and  $rf'(r) > 0$  on  $[0, 4]$ , so  $h'(r) > 0$   
 $\therefore h(r)$  is inc.

The approx in part b is overestimate of total mass b/c  $rf(r)$  is inc.

1pt: answer w/ explanation

Response for question 1(d)

$g(k) = \frac{1}{b-a} \int_a^b g(r) dr \dots \dots \dots \text{☺}$

$2 - 16(\cos 1.57\sqrt{k})^3 = \frac{1}{4-1} \int_1^4 g(r) dr$

$k = 2.497$

1pt: definite integral

1pt: average value

1pt: answer

☺  
 no soln

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\begin{aligned} \text{speed} &= |v(1.2)| \\ &= \sqrt{((1.2-1)e^{(1.2)^2})^2 + (\sin(1.2^{1.25}))^2} \\ &= 1.271 \end{aligned}$$

1 pt: speed

$$\begin{aligned} a(1.2) &= v'(1.2) \\ &= \langle x''(1.2), y''(1.2) \rangle \\ &= \langle 6.247, 0.405 \rangle \end{aligned}$$

1 pt: acceleration  
vector

Response for question 2(b)

$$v(t) = \langle x'(t), y'(t) \rangle$$

$$\begin{aligned} \text{total distance} &= \int_0^1 |v(t)| dt \\ &= \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2} dt \\ &= 1.010 \end{aligned}$$

1 pt: integrand  
1 pt: answer

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

Farthest left  $\rightarrow$  abs min  $x(t)$  only 😊

$$x'(t) = (t-1)e^{t^2} = 0$$
$$t = 1$$

$$x(1) = x(0) + \int_0^1 x'(t) dt$$
$$= -2 + \int_0^1 x'(t) dt$$
$$= -2.604$$

lpt:  $x'(t) = 0$   
lpt: leftmost point  
@  $t = 1$   
lpt: one coordinate  
of leftmost point

Particle is furthest left @  $t = 1$

b/c  $x'(t) < 0$  on  $(0, 1)$  and  $x'(t) > 0$  on  $(1, \infty)$  ... 😊

explain this,  
so no need  
to check endpoints

$$y(1) = y(0) + \int_0^1 y'(t) dt$$
$$= 5 + \int_0^1 y'(t) dt$$
$$= 5.410$$

Particle is furthest left @  $\langle -2.604, 5.410 \rangle$

lpt: leftmost  
point

$x'(t) > 0$  on  $(1, \infty)$ ,  $\therefore$ , particle moves right on  $(1, \infty)$

$$\int_0^{\infty} x'(t) dt = \infty,$$

$\therefore$  particle does not have a pt.  
furthest to the right

lpt: explanation