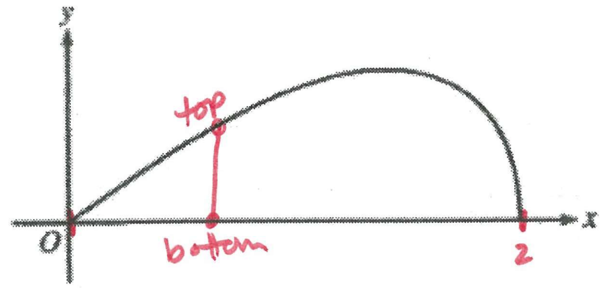
Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$\text{Area} = \int_0^2 6x\sqrt{4-x^2}\,dx$$

$6x\sqrt{4-x^2} = 0$

$6x = 0 \quad \sqrt{4-x^2} = 0$

$x = 0 \quad 4 - x^2 = 0$

$4 = x^2$

$\pm 2 = x$

1pt: integrand

$u = 4 - x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2x} = dx$

$u(2) = 0$

$u(0) = 4$

$$= \int_4^0 6x \cdot \sqrt{u} \cdot \frac{du}{-2x}$$

$$= -3\int_4^0 u^{1/2}\,du$$

$$= -3\left(\frac{2}{3}u^{3/2}\right)\Big|_4^0$$

1pt: antiderivative

$$= -3\left(\frac{2}{3}\right)\left(0^{3/2} - 4^{3/2}\right)$$ ← ok to stop here

$$= -2(-8)$$

$$= 16$$

1pt: answer

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

Largest → max,
max when $dy/dx = 0$

$$\frac{dy}{dx} = \frac{c(4-2x^2)}{\sqrt{4-x^2}}$$

$$0 = \frac{c(4-2x^2)}{\sqrt{4-x^2}}$$

$$0 = c(4-2x^2)$$

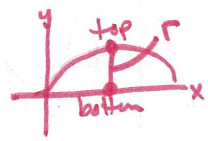
$$0 = 4-2x^2$$

$$2x^2 = 4$$

$$x^2 = 2$$

$x = \pm\sqrt{2}$ ← but only positive radius

$$x = \sqrt{2}$$



$$r = cx\sqrt{4-x^2}$$

$$1.2 = c(\sqrt{2})(\sqrt{4-(\sqrt{2})^2})$$

$$1.2 = \sqrt{2}\,c\,(\sqrt{4-2})$$

$1.2 = \sqrt{2}\cdot\sqrt{2}\,c$ → or $c = \frac{1.2}{\sqrt{2}\sqrt{2}}$

$$1.2 = 2c$$

$$0.6 = c$$

ok to stop here

$dy/dx = 0$
1pt: ~~[illegible]~~
1pt: answer

Response for question 3(c)

$$\text{Volume} = \pi\int_0^2 (cx\sqrt{4-x^2})^2\,dx$$

$$2\pi = \pi\int_0^2 c^2x^2(4-x^2)\,dx$$

$$2 = c^2\int_0^2 (4x^2-x^4)\,dx$$

$$2 = c^2\left(\frac{4}{3}x^3 - \frac{1}{5}x^5\right)\Big|_0^2$$

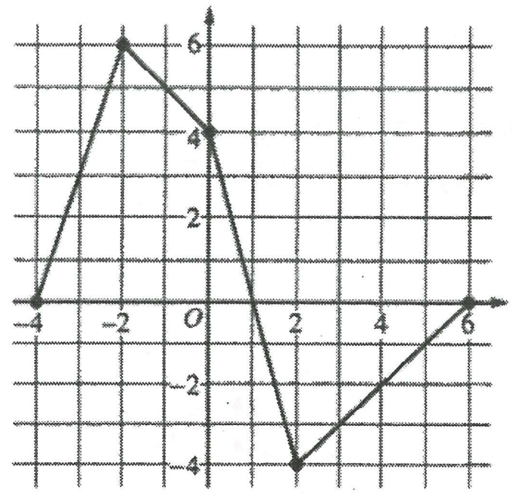
$$2 = c^2\left(\frac{4}{3}(2)^3 - \frac{1}{5}(2)^5\right)$$

$$\frac{2}{\frac{4}{3}(2)^3 - \frac{1}{5}(2)^5} = c^2$$

ok to stop here → $\sqrt{\dfrac{2}{\frac{4}{3}(2)^3-\frac{1}{5}(2)^5}} = c$ or $c = \sqrt{\dfrac{15}{32}}$

1pt: form of integrand
1pt: limits and constant
1pt: antiderivative
1pt: answer

Answer QUESTION 4 parts (a) and (b) on this page.



Graph of f

1pt: $G'(x) = f(x)$

Response for question 4(a)

G concave up → $G'' > 0$

$G(x) = \int_0^x f(t)\,dt$

$G'(x) = f(x)$

$G''(x) = f'(x)$

$f' = 0$ or DNE @ $x = -2, 0, 2$

f': + | − | − | + at −2, 0, 2

G is concave up on $(-4,-2) \cup (2,6)$ b/c $G'' > 0$ on those intervals

1pt: answer w/ reason

Response for question 4(b)

$P(x) = G(x) f(x)$

$P'(x) = f(x) G'(x) + G(x) f'(x)$

$P'(3) = f(3) G'(3) + G(3) f'(3)$

$= -3 \cdot -3 + -3.5 \cdot 1$ ← ok to stop here

$= 9 - 3.5$

$= 5.5$

$G(3) = \int_0^3 f(t)\,dt$

$= -3.5$

1pt: product rule

1pt: $G(3)$ or $G'(3)$

1pt: answer

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x\to 2} \frac{G(x)}{x^2-2x} = \frac{\lim_{x\to 2} G(x) \to 0}{\lim_{x\to 2}(x^2-2x) \to 0}$$

$$= \frac{\lim_{x\to 2} G'(x)}{\lim_{x\to 2}(2x-2)}$$

$G'(x)$ cont b/c f cont,
∴ $G(x)$ cont since $G(x)$ is diff'able

$$= \frac{G'(2)}{2}$$

$$= \frac{f(2)}{2}$$

$$= \frac{-4}{2}$$ ← ok to stop here

$$= -2$$

1pt: uses L'Hôpital's rule

1pt: answer w/ justification

Response for question 4(d)

$$\text{avg rate of change of } G = \frac{G(b)-G(a)}{b-a}$$

$$G'(c) = \frac{G(2)-G(-4)}{2--4}$$

$$= \frac{0-(-16)}{2--4}$$

$$= \frac{16}{6}$$

$$= 8/3$$

$$G(-4) = \int_0^{-4} f(t)\,dt = -\int_{-4}^{0} f(t)\,dt = -16$$

1pt: average rate of change

1pt: answer w/ reason

$f(x)$ is cont, so $G'(x) = f(x)$ is cont on $[-4,2]$

∴, G is diff'able on $(-4,2)$ and G is cont on $[-4,2]$

∴, yes, MVT guarantees a value c on $(-4, 2)$ such that $G'(c) = 8/3$.

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$f'(1) = 4 \cdot 1 \cdot \ln 1$
$= 0$

$f(x) \approx f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2$

$\approx 4 + 0(x-1) + \frac{4}{2}(x-1)^2$

1pt: polynomial

$f(2) \approx 4 + 2(1)^2$ ← ok to stop here

1pt: approximation

≈ 6

Response for question 5(b)

$\Delta x = \frac{2-1}{2} = \frac{1}{2}$

	$\frac{dy}{dx}$	$\frac{dy}{dx}\Delta x$	$\Delta y + y_1$
(1,4)	0	0	4+0 = 4
(1.5, 4)	$4(1.5)\ln 1.5$	$\frac{1}{2}(4)(1.5)\ln 1.5$	$4 + \frac{1}{2}(4)(1.5)\ln 1.5$

1pt: Euler's w/ 2 step

$f(2) \approx 4 + \frac{1}{2}(4)(1.5)\ln 1.5$ ← ok to not simplify

1pt: answer

OR

$f(2) \approx 4 + 3\ln 1.5$

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = yx\ln x$$

LIPET

$$\int \frac{1}{y}\,dy = \int x \ln x \, dx$$

$u = \ln x \quad dV = x$

$du = \frac{1}{x}dx \quad V = \frac{1}{2}x^2$

$uv - \int v\,du$

$$\ln|y| = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$$

$$\ln|y| = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx$$

$$\ln|y| = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

$$\ln 4 = -\frac{1}{4} + C$$

$$\frac{1}{4} + \ln 4 = C$$

$$\ln|y| = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + \frac{1}{4} + \ln 4$$

$$|y| = e^{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + \frac{1}{4} + \ln 4}$$

$$y = e^{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + \frac{1}{4} + \ln 4}$$

1pt: separate variables

1pt: antiderivative of $x \ln x$

1pt: antiderive $\frac{1}{y}$

1pt: +C and initial condition

1pt: solves for y

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$\sum_{n=0}^{\infty} \frac{1}{e^n}$

1pt: conditions

$\frac{1}{e^x}$ is positive, continuous, dec on $[0,\infty)$

1pt: improper integral

$\int_0^\infty \frac{1}{e^x}\,dx = \int_0^\infty e^{-x}\,dx$

$= \lim_{a\to\infty} \int_0^a e^{-x}\,dx$

$= \lim_{a\to\infty} (-e^{-x})\Big|_0^a$

1pt: evaluation

$= \lim_{a\to\infty} (-e^{-a} - (-1))$

$= 1$, $\therefore$, $\int_0^\infty e^{-x}\,dx$ converges

so, by Integral Test, $\sum_{n=0}^{\infty} \frac{1}{e^n}$ also converges

Response for question 6(b)

1pt: sets up limit comparison

$\sum_{n=0}^{\infty} \left|\frac{(-1)^n}{2e^n+3}\right| = \sum_{n=0}^{\infty} \frac{1}{2e^n+3}$ compares to $\sum_{n=0}^{\infty} \frac{1}{e^n}$

1pt: explanation

$\lim_{n\to\infty} \left(\frac{1}{2e^n+3} \cdot \frac{e^n}{1}\right) = \frac{1}{2}$

> 0 and $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges (from part a)

$\therefore$, $\sum \frac{1}{2e^n+3}$ converges by limit comparison test

and, thus, $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n+3}$ converges absolutely.

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{n+1}}{2e^{n+1}+3}\cdot\frac{2e^{n}+3}{(-1)^{n}x^{n}}\right|$$

1pt: sets up ratio

$$=\lim_{n\to\infty}\left|\frac{(-1)^{1}(2e^{n}+3)x}{2e^{n+1}+3}\right|$$

1pt: computes limit of ratio

$$=\lim_{n\to\infty}\left|\frac{2e^{n}+3}{2e^{n+1}+3}x\right|$$

$$=\frac{1}{e}|x|$$

$$\frac{1}{e}|x|<1$$

$$|x|<e$$

1pt: answer

← radius of convergence $= e$

Response for question 6(d)

alt series w/ terms decreasing in abs value to 0.

$|\text{error}| \le |1^{st}\text{ unused term}|$

$$\le\left|\frac{(-1)^{2}x^{2}}{2e^{2}+3}\right|$$

1pt: answer

$$\left|\underset{@\,x=1}{\text{error}}\right| \le \frac{1}{2e^{2}+3}$$

upper bound is $\frac{1}{2e^{2}+3}$