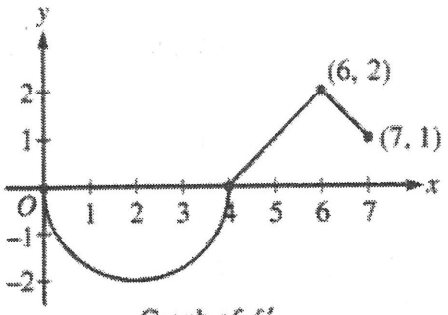
2022

AP CollegeBoard

AP® Calculus BC

Free-Response Questions

Answer QUESTION 3 parts (a) and (b) on this page.



Graph of f'

1 pt: area of either $\int_0^4 f'(x)\,dx$ or $\int_4^0 f'(x)\,dx$

Response for question 3(a)

$f(b) = \text{initial condition} + \int_a^b f'(x)\,dx$ ☺

$f(0) = f(4) + \int_4^0 f'(x)\,dx$

$= 3 - \int_0^4 f'(x)\,dx$

$= 3 - \left(-\frac{1}{2}\pi(2)^2\right)$ ← ok to stop here

$= 3 + 2\pi$

$f(5) = f(4) + \int_4^5 f'(x)\,dx$

$= 3 + \frac{1}{2}(1)(1)$ ← ok to stop here

$= \frac{7}{2}$ or 3.5

1 pt: $f(0)$

1 pt: $f(5)$

Response for question 3(b)

pt of inf for $f \rightarrow f''$ changes signs
or slope of f' changes signs
or f' inc to dec, f' dec to inc ☺

slopes of f' $= f''$: − + + − at $x = 2, 4, 6, 7$

f has inf pts @ $x = 2$ and $x = 6$

b/c slopes of f' change signs @ $x = 2$ and $x = 6$.

1 pt: answer

1 pt: reason

Answer QUESTION 3 parts (c) and (d) on this page.

Response for question 3(c)

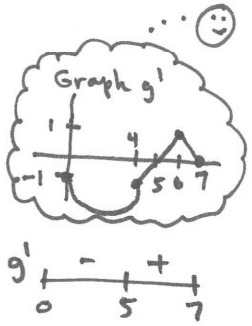
$g(x) = f(x) - x$

$g'(x) = f'(x) - 1$

$0 = f'(x) - 1$

$1 = f'(x)$

$x = 5, x = 7$



1pt: interval (ok as (0,5) or [0,5])

g is dec on $[0,5)$ b/c $g' < 0$ on $[0,5)$

1pt: reason

Response for question 3(d) abs min ⇒ candidates test

$g(0) = f(0) - 0$

$= 3 + 2\pi$

$g(5) = f(5) - 5$

$= 3.5 - 5$

$= -1.5$

1pt: considers $g'(x) = 0$

$g(7) = f(7) - 7$

$= 3 + \int_5^7 f'(x)\,dx - 7$

$= 3 + \frac{1}{2}(2)(2)^{+1} + \frac{1}{2}(1)(1) - 7$

$= -4 + 2 + 1 + \frac{1}{2}$

$= -\frac{1}{2}$

1pt: answer w/ justification

abs min value of g is -1.5

Answer QUESTION 4 parts (a) and (b) on this page.

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	−6.1	−5.0	−4.4	−3.8	−3.5

Response for question 4(a)

$$r''(8.5) = \frac{r'(10) - r'(7)}{10 - 7}$$

$$= \frac{-3.8 - (-4.4)}{10 - 7}$$

$$= 0.2 \text{ cm/day}^2$$

1pt: $r''(8.5)$ w/ work

1pt: units

Response for question 4(b)

IVT... ☺

$r'(t)$ is cont on $[0,3]$ b/c r is twice-diff'able

$r'(0) = -6.1$
$r'(3) = -5.6$
} -6 is btn. -6.1 and -5.0

∴ there is a time t on $(0,3)$ for which $r'(t) = -6$.

1pt: betweenness $r'(0) < -6 < r'(3)$

1pt: conclusion using IVT.

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\int_0^{12} r'(t)\,dt \approx 2(-3.5) + 3(-3.8) + 4(-4.4) + 3(-5.0)$$

$$\approx -51$$

← ok to stop here

1pt: form of right sum

1pt: answer

Response for question 4(d)

$\frac{dh}{dt} = -2$ cm/day

$r = 100$ cm

$h = 50$ cm

$\frac{dV}{dt} = ?$ @ $t = 3$ days

$V = \frac{1}{3}\pi r^2 h$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(h \cdot 2r\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$$

$$= \frac{1}{3}\pi\left(50 \cdot 2 \cdot 100(-5) + 100^2(-2)\right)$$

← ok to stop here

$$= \frac{-70000}{3}\pi$$

1pt: product rule

1pt: chain rule

1pt: answer

Answer QUESTION 5 part (a) on this page.

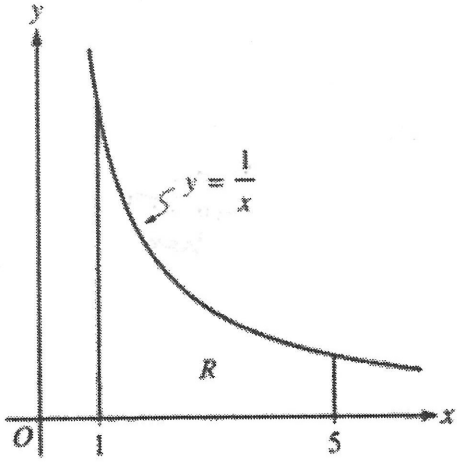


Figure 1

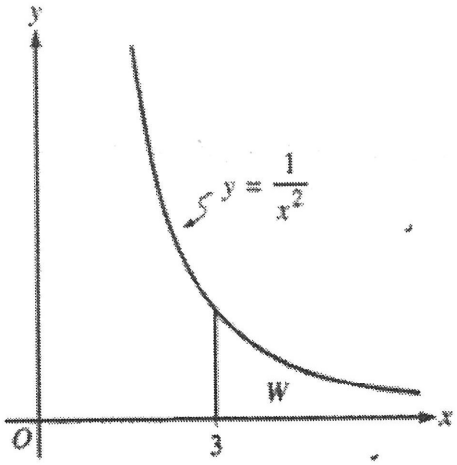


Figure 2

Response for question 5(a)

Area of R $= \int_1^5 \frac{1}{x}\,dx$

$= \ln|x|\,\Big|_1^5$

$= \ln 5 - \ln 1$ ← ok to stop here

$= \ln 5$

1 pt : integral

1 pt : answer

Answer QUESTION 5 parts (b) and (c) on this page.

Response for question 5(b)

rectangle

Area rectangle $= xe^{x/5}$

1 pt: definite integral

1 pt: u and dv

$$\text{Volume} = \int_1^5 xe^{x/5}\,dx$$

	u	dv
+	x	$e^{x/5}$
−	1	$5e^{x/5}$
	0	$25e^{x/5}$

1 pt: correct by parts

$$= (5xe^{x/5} - 25e^{x/5})\Big|_1^5$$

$$= 5\cdot 5e^{5/5} - 25e^{5/5} - (5e^{1/5} - 25e^{1/5})$$ ← ok to stop here

1 pt: answer

$$= 25e - 25e - 5e^{1/5} + 25e^{1/5}$$

$$= 20e^{1/5}$$

Response for question 5(c)

$$\text{Volume} = \pi\int_3^\infty \left(\frac{1}{x^2}\right)^2 dx$$

1 pt: improper integral

$$= \pi\int_3^\infty x^{-4}\,dx$$

$$= \pi \lim_{a\to\infty}\int_3^a x^{-4}\,dx$$

1 pt: antiderivative

$$= \pi \lim_{a\to\infty}\left(-\tfrac{1}{3}x^{-3}\right)\Big|_3^a$$

$$= \pi \lim_{a\to\infty}\left(-\frac{1}{3a^3} - \frac{-1}{3(3)^3}\right)$$

1 pt: answer

$$= \pi\left(\frac{1}{3(3)^3}\right)$$ ← ok to stop here

$$= \frac{\pi}{81}$$

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

ratio test

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2(n+1)+1}}{2(n+1)+1}\cdot\frac{2n+1}{(-1)^n x^{2n+1}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(-1)x^2(2n+1)}{2n+3}\right|$$

$$=|-1x^2|$$

$$=|x^2|<1$$

$$\sqrt{x^2}<\sqrt{1}$$

$$|x|<1$$

$$-1<x<1$$

$-1 \le x \le 1$ is interval of convergence

endpts:

$x=-1$, $\sum_{n=0}^{\infty}\frac{(-1)^n(-1)^{2n+1}}{2n+1}$

$=\sum_{n=0}^{\infty}\frac{(-1)^{3n+1}}{2n+1}$

$=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2n+1}$

Converges by alt. series test

$\frac{1}{2n+1}>0$

$\frac{1}{2n+1}>\frac{1}{2n+3}$

$\lim_{n\to\infty}\frac{1}{2n+1}=0$

$\therefore x=-1$ is included

$x=1$, $\sum_{n=0}^{\infty}\frac{(-1)^n(1)^{2n+1}}{2n+1}$

$=\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$

also converges b/c alt series whose terms decrease in value to 0.

$\therefore x=1$ is included

1 pt: sets up ratio

1 pt: identifies $-1<x<1$

1 pt: considers both endpts

1 pt: analysis and interval of convergence

Response for question 6(b)

$$\left|f\left(\tfrac{1}{2}\right)-\tfrac{1}{2}\right| \le \left|\frac{(-\frac{1}{2})^3}{3}\right|$$

$$<\frac{1}{10}$$

1st unused term

alt series w/ individual terms that dec in abs value to zero

1 pt: uses 2nd term

1 pt: justification

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots$$

$$f'(x) = 1 - x^2 + x^4 - x^6 + \cdots + \frac{(-1)^n (2n+1) x^{2n}}{2n+1} + \cdots$$

$$= 1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots$$

1 pt: 1st 4 terms
1 pt: general term

Response for question 6(d)

$f'(x)$ is geo series $\rightarrow a = 1$
$r = -x^2$

$$f'(x) = \frac{1}{1-(-x^2)}$$

$$= \frac{1}{1+x^2}$$

$$f'\left(\frac{1}{6}\right) = \frac{1}{1+\left(\frac{1}{6}\right)^2} \leftarrow \text{ok to stop here}$$

$$= \frac{1}{1+\frac{1}{36}}$$

$$= \frac{1}{\frac{37}{36}}$$

$$= \frac{36}{37}$$

1 pt: answer