CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

   (a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

   (b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

   (c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

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2. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t > 0$, where

$$\frac{dx}{dt} = \left(\frac{6}{t} - 3\right)^{1/3} \text{ and } \frac{dy}{dt} = te^{-t}.$$

At time $t = 3$, the particle is at the point $(5, 4)$.

(a) Find the speed of the particle at time $t = 3$.

(b) Write an equation for the line tangent to the path of the particle at time $t = 3$.

(c) Is there a time t at which the particle is farthest to the right? If yes, explain why and give the value of t and the x-coordinate of the position of the particle at that time. If no, explain why not.

(d) Describe the behavior of the path of the particle as t increases without bound.

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t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

3. The temperature, in degrees Celsius $(°C)$, of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

(a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

(b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

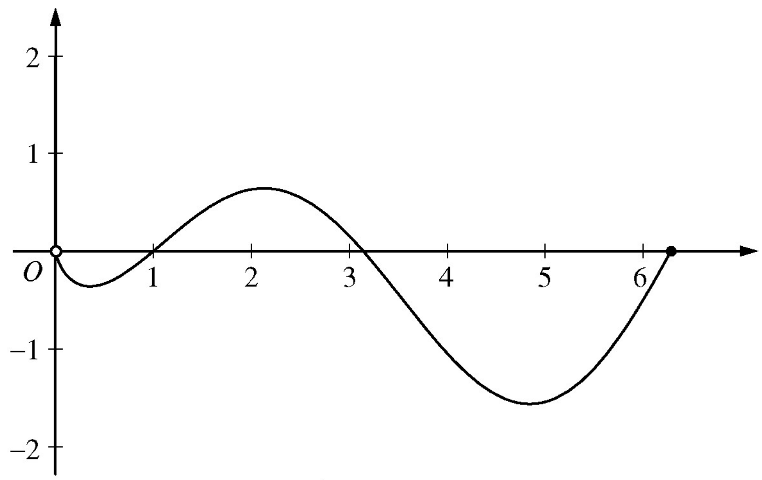
(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

   (a) Find $g(1)$ and $g'(1)$.

   (b) On what intervals, if any, is g increasing? Justify your answer.

   (c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

   (d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

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5. Let f be the function satisfying $f'(x) = 4x - 2xf(x)$ for all real numbers x, with $f(0) = 5$ and $\lim_{x \to \infty} f(x) = 2$.

(a) Find the value of $\int_0^\infty (4x - 2xf(x))\, dx$. Show the work that leads to your answer.

(b) Use Euler's method to approximate $f(-1)$, starting at $x = 0$, with two steps of equal size.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 4x - 2xy$ with the initial condition $f(0) = 5$.

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6. The function g is continuous for all real numbers x and is defined by

$$g(x) = \frac{\cos(2x) - 1}{x^2} \text{ for } x \neq 0.$$

(a) Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

(b) Let f be the function given by $f(x) = \cos(2x)$. Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(c) Use your answer from part (b) to write the first three nonzero terms and the general term of the Taylor series for g about $x = 0$.

(d) Determine whether g has a relative minimum, a relative maximum, or neither at $x = 0$. Justify your answer.

STOP

END OF EXAM
