

# Calculus BC Boot Camp Unit 1: Limits, Continuity, and Derivatives

<u>Date</u>	<u>Lesson</u>	<u>HW Assignment/Lesson Practice</u>
1-Jun	Limits	HW1 -- Limits Handout
2-Jun	Continuity	HW2 -- Continuity Handout
4-Aug	Tangent Lines & Definition of the Derivative	HW3 -- Rates of Change, Tangent Lines, & Definition of Derivative Handout
6-Aug	Rules for Differentiation	HW4 -- Basic Rules Handout
7-Aug	Rules for Differentiation	HW5 -- More Rules Handout <b>Study for Test in 1st week of school</b>

## Calculus A Essential Knowledge (to be learned prior to start of Calculus BC)

### Limits

<b>LIM-1</b>	<b>LIM-2</b>
Reasoning with definitions, theorems, and properties can be used to justify claims about limits.	Reasoning with definitions, theorems, and properties can be used to justify claims about continuity.
<p><b>LEARNING OBJECTIVE</b></p> <p><b>LIM-1.A</b> Represent limits analytically using correct notation.</p> <p><b>ESSENTIAL KNOWLEDGE</b></p> <p><b>LIM-1.A.1</b> Given a function <math>f</math>, the limit of <math>f(x)</math> as <math>x</math> approaches <math>c</math> is a real number <math>R</math> if <math>f(x)</math> can be made arbitrarily close to <math>R</math> by taking <math>x</math> sufficiently close to <math>c</math> (but not equal to <math>c</math>). If the limit exists and is a real number, then the common notation is <math>\lim_{x \rightarrow c} f(x) = R</math>.</p> <p><b>X EXCLUSION STATEMENT</b> <i>The epsilon-delta definition of a limit is not assessed on the AP Calculus AB or BC Exam. However, teachers may include this topic in the course if time permits.</i></p>	<p><b>LEARNING OBJECTIVE</b></p> <p><b>LIM-2.D</b> Interpret the behavior of functions using limits involving infinity.</p> <p><b>ESSENTIAL KNOWLEDGE</b></p> <p><b>LIM-2.D.1</b> The concept of a limit can be extended to include infinite limits.</p> <p><b>LIM-2.D.2</b> Asymptotic and unbounded behavior of functions can be described and explained using limits.</p>
<p><b>LIM-1.B</b> Interpret limits expressed in analytic notation.</p> <p><b>LIM-1.C</b> Estimate limits of functions.</p>	<p><b>LIM-2.D</b> Interpret the behavior of functions using limits involving infinity.</p> <p><b>LIM-2.D.3</b> The concept of a limit can be extended to include limits at infinity.</p> <p><b>LIM-2.D.4</b> Limits at infinity describe end behavior.</p> <p><b>LIM-2.D.5</b> Relative magnitudes of functions and their rates of change can be compared using limits.</p>
<p><b>LIM-1.C</b> Estimate limits of functions.</p> <p><b>LIM-1.D</b> Determine the limits of functions using limit theorems.</p> <p><b>LIM-1.E</b> Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.</p>	<p><b>LIM-1.C.1</b> The concept of a limit includes one sided limits.</p> <p><b>LIM-1.C.2</b> Graphical information about a function can be used to estimate limits.</p> <p><b>LIM-1.C.3</b> Because of issues of scale, graphical representations of functions may miss important function behavior.</p> <p><b>LIM-1.C.4</b> A limit might not exist for some functions at particular values of <math>x</math>. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.</p> <p><b>LIM-1.C.5</b> Numerical information can be used to estimate limits.</p> <p><b>LIM-1.D.1</b> One-sided limits can be determined analytically or graphically.</p> <p><b>LIM-1.D.2</b> Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems.</p> <p><b>LIM-1.E.1</b> It may be necessary or helpful to rearrange expressions into equivalent forms before evaluating limits.</p>

# Calculus BC Boot Camp Unit 1: Limits, Continuity, and Derivatives

## Continuity

<p><b>LIM-2</b> Reasoning with definitions, theorems, and properties can be used to justify claims about continuity.</p>	
<p><b>LEARNING OBJECTIVE</b></p> <p><b>LIM-2.A</b> Justify conclusions about continuity at a point using the definition.</p>	<p><b>ESSENTIAL KNOWLEDGE</b></p> <p><b>LIM-2.A.1</b> Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.</p>
<p><b>LIM-2.A</b> Justify conclusions about continuity at a point using the definition.</p>	<p><b>LIM-2.A.2</b> A function <math>f</math> is continuous at <math>x = c</math> provided that <math>f(c)</math> exists, <math>\lim_{x \rightarrow c} f(x)</math> exists, and <math>\lim_{x \rightarrow c} f(x) = f(c)</math>.</p>
<p><b>LIM-2.B</b> Determine intervals over which a function is continuous.</p>	<p><b>LIM-2.B.1</b> A function is continuous on an interval if the function is continuous at each point in the interval.</p> <p><b>LIM-2.B.2</b> Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains.</p>
<p><b>LIM-2.C</b> Determine values of <math>x</math> or solve for parameters that make discontinuous functions continuous, if possible.</p>	<p><b>LIM-2.C.1</b> If the limit of a function exists at a discontinuity in its graph, then it is possible to remove the discontinuity by defining or redefining the value of the function at that point, so it equals the value of the limit of the function as <math>x</math> approaches that point.</p> <p><b>LIM-2.C.2</b> In order for a piecewise-defined function to be continuous at a boundary to the partition of its domain, the value of the expression defining the function on one side of the boundary must equal the value of the expression defining the other side of the boundary, as well as the value of the function at the boundary.</p>

## Derivatives

<p><b>CHA-2</b> Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.</p>	
<p><b>LEARNING OBJECTIVE</b></p> <p><b>CHA-2.A</b> Determine average rates of change using difference quotients.</p>	<p><b>ESSENTIAL KNOWLEDGE</b></p> <p><b>CHA-2.A.1</b> The difference quotients <math>\frac{f(a+h) - f(a)}{h}</math> and <math>\frac{f(x) - f(a)}{x - a}</math> express the average rate of change of a function over an interval.</p>
<p><b>CHA-2.B</b> Represent the derivative of a function as the limit of a difference quotient.</p>	<p><b>CHA-2.B.1</b> The instantaneous rate of change of a function at <math>x = a</math> can be expressed by <math>\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}</math> or <math>\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}</math>, provided the limit exists. These are equivalent forms of the definition of the derivative and are denoted <math>f'(a)</math>.</p>
<p><b>CHA-2.B</b> Represent the derivative of a function as the limit of a difference quotient.</p>	<p><b>CHA-2.B.2</b> The derivative of <math>f</math> is the function whose value at <math>x</math> is <math>\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}</math>, provided this limit exists.</p> <p><b>CHA-2.B.3</b> For <math>y = f(x)</math>, notations for the derivative include <math>\frac{dy}{dx}</math>, <math>f'(x)</math>, and <math>y'</math>.</p> <p><b>CHA-2.B.4</b> The derivative can be represented graphically, numerically, analytically, and verbally.</p>
<p><b>CHA-2.C</b> Determine the equation of a line tangent to a curve at a given point.</p>	<p><b>CHA-2.C.1</b> The derivative of a function at a point is the slope of the line tangent to a graph of the function at that point.</p>
<p><b>CHA-2.D</b> Estimate derivatives.</p>	<p><b>CHA-2.D.1</b> The derivative at a point can be estimated from information given in tables or graphs.</p> <p><b>CHA-2.D.2</b> Technology can be used to calculate or estimate the value of a derivative of a function at a point.</p>

## Derivatives (continued)

<p><b>FUN-3</b> Recognizing opportunities to apply derivative rules can simplify differentiation.</p>	
<p><b>LEARNING OBJECTIVE</b></p> <p><b>FUN-3.A</b> Calculate derivatives of familiar functions.</p>	<p><b>ESSENTIAL KNOWLEDGE</b></p> <p><b>FUN-3.A.1</b> Direct application of the definition of the derivative and specific rules can be used to calculate the derivative for functions of the form <math>f(x) = x^r</math>.</p>
<p><b>FUN-3.A</b> Calculate derivatives of familiar functions.</p>	<p><b>FUN-3.A.2</b> Sums, differences, and constant multiples of functions can be differentiated using derivative rules.</p> <p><b>FUN-3.A.3</b> The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions.</p>
<p><b>FUN-3.A</b> Calculate derivatives of familiar functions.</p>	<p><b>FUN-3.A.4</b> Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions.</p>
<p><b>FUN-3.B</b> Calculate derivatives of products and quotients of differentiable functions.</p>	<p><b>FUN-3.B.1</b> Derivatives of products of differentiable functions can be found using the product rule.</p>
<p><b>FUN-3.B</b> Calculate derivatives of products and quotients of differentiable functions.</p>	<p><b>FUN-3.B.2</b> Derivatives of quotients of differentiable functions can be found using the quotient rule.</p>