CALCULUS AB FINAL EXAM SEMESTER 1 REVIEW

PART I NON-CALCULATOR: MULTIPLE-CHOICE
NO calculator may be used on this part of the review.

1. f is continuous for $a \le x \le b$ but not differentiable for some c such that $a < c < b$. Which of the following could be true?

(A) $x = c$ is a vertical asymptote of the graph of f.

(B) $\lim_{x \to c} f(x) \neq f(c)$

(C) The graph of f has a cusp at $x = c$.

(D) $f(c)$ is undefined.

(E) None of the above

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2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

(A) $-\frac{3x+2y}{y^2}$

(B) $-\frac{3x+y}{x+y}$

(C) $\frac{1-3x-y}{x+y}$

(D) $-\frac{3x}{1+y}$

(E) $-\frac{3x}{x+y}$

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x	1	2	3
$f(x)$	2	k	4

3. The function f is continuous on the closed interval [1,3] and has the values in the table given above. The equation $f(x) = \frac{5}{4}$ must have at least two solutions in the interval [1,3] if k =

(A) $\frac{1}{4}$

(B) $\frac{3}{2}$

(C) 2

(D) $\frac{9}{4}$

(E) 3

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4. If $\frac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx} f(h(x)) =$

(A) $g(x^2)$

(B) $2xg(x)$

(C) $g'(x)$

(D) $2xg(x^2)$

(E) $x^2 g(x^2)$

5. If $f(x) = \tan 3x$, then $f'\left(\frac{\pi}{9}\right) =$

(A) $\frac{4}{3}$

(B) 4

(C) 6

(D) 12

(E) $6\sqrt{3}$

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6. The function f is given by $f(x) = -x^6 + x^3 - 2$. On which of the following intervals is f decreasing?

(A) $(-\infty, 0)$

(B) $\left(-\infty, -\sqrt[3]{\frac{1}{2}}\right)$

(C) $\left(0, \sqrt[3]{\frac{1}{2}}\right)$

(D) $(0, \infty)$

(E) $\left(\sqrt[3]{\frac{1}{2}}, \infty\right)$

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7. An equation of the line tangent to the graph of $y = 3x - \cos x$ at $x = 0$ is

(A) $2x - y = 0$

(B) $2x - y = 1$

(C) $3x - y = -1$

(D) $3x - y = 1$

(E) $4x - y = 0$

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8. The graph of a twice-differentiable function f is shown below. Which of the following is true?

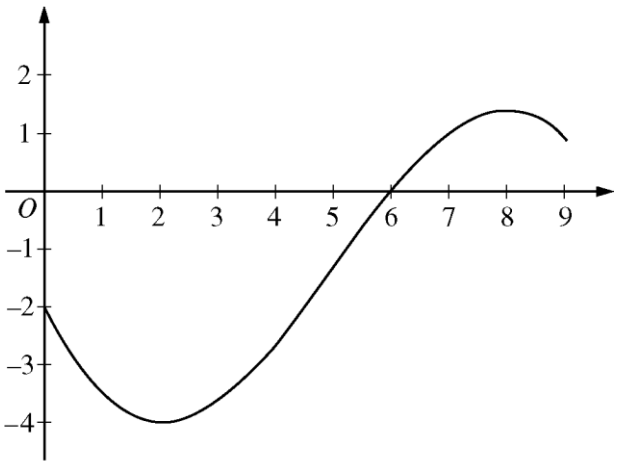
(A) $f(6) < f'(6) < f''(6)$

(B) $f(6) < f''(6) < f'(6)$

(C) $f'(6) < f(6) < f''(6)$

(D) $f''(6) < f(6) < f'(6)$

(E) $f''(6) < f'(6) < f(6)$



Graph of f

9. If $f(x) = \cos e^{2x}$, then $f'(x) =$

(A) $\sin e^{2x}$

(B) $2\sin e^{2x}$

(C) $-\sin e^{2x}$

(D) $-2\sin e^{2x}$

(E) $-2e^{2x}\sin e^{2x}$

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10. Determine the value of *c* so that *f(x)* continuous on the entire real line when $f(x) = \begin{cases} x+3\ , x \le 2 \\ cx+6\ , x > 2 \end{cases}$

(A) 0

(B) -2

(C) 1

(D) $-\dfrac{1}{2}$

(E) None of these

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11. Find $\frac{dy}{dx}$ if: $x^2 + 3xy + y^3 = 10$

(A) $-\dfrac{2x+3y}{3x+3y^2}$

(B) $\dfrac{2x-3y}{3x+3y^2}$

(C) $-\dfrac{x+y}{x+y^2}$

(D) $\dfrac{x-y}{x+y^2}$

(E) None of these

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12. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top moving down the wall when the base of the ladder is 7 feet.

(A) $\frac{7}{12}$ ft/min

(B) $-\frac{7}{12}$ ft/min

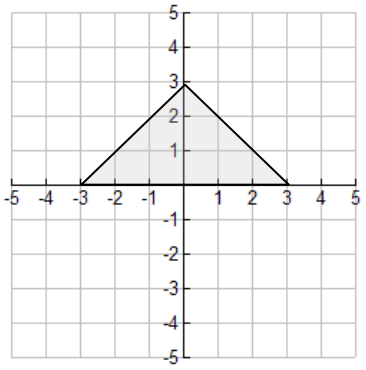
(C) $\frac{12}{7}$ ft/min

(D) $-\frac{12}{7}$ ft/min

(E) None of these

13. Set up a definite integral that yields the area of the region.

$$f(x) = 3 - |x|$$



(A) $\int_3^3 (3 - x)\, dx$

(B) $\int_0^3 |x|\, dx$

(C) $\int_3^{-3} (3 - |x|) dx$

(D) $\int_{-3}^3 (3 - |x|) dx$

(E) $\int_3^0 |x|\, dx$

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14. A particle moves along the *x*-axis so that its position at time *t* is given by $x(t) = t^2 - 7t + 12$. For what value of *t* is the velocity of the particle zero?

(A) 2.5

(B) 3

(C) 3.5

(D) 4

(E) 4.5

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15. If $f(x) = \sqrt{e^x}$, then $f'(\ln 2) =$

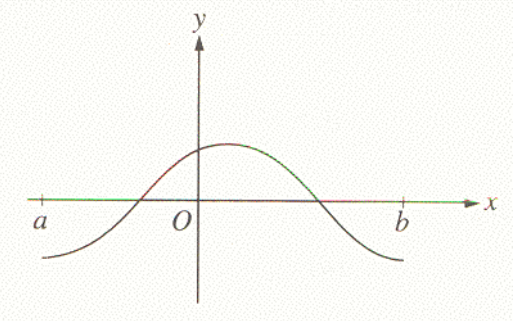
(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

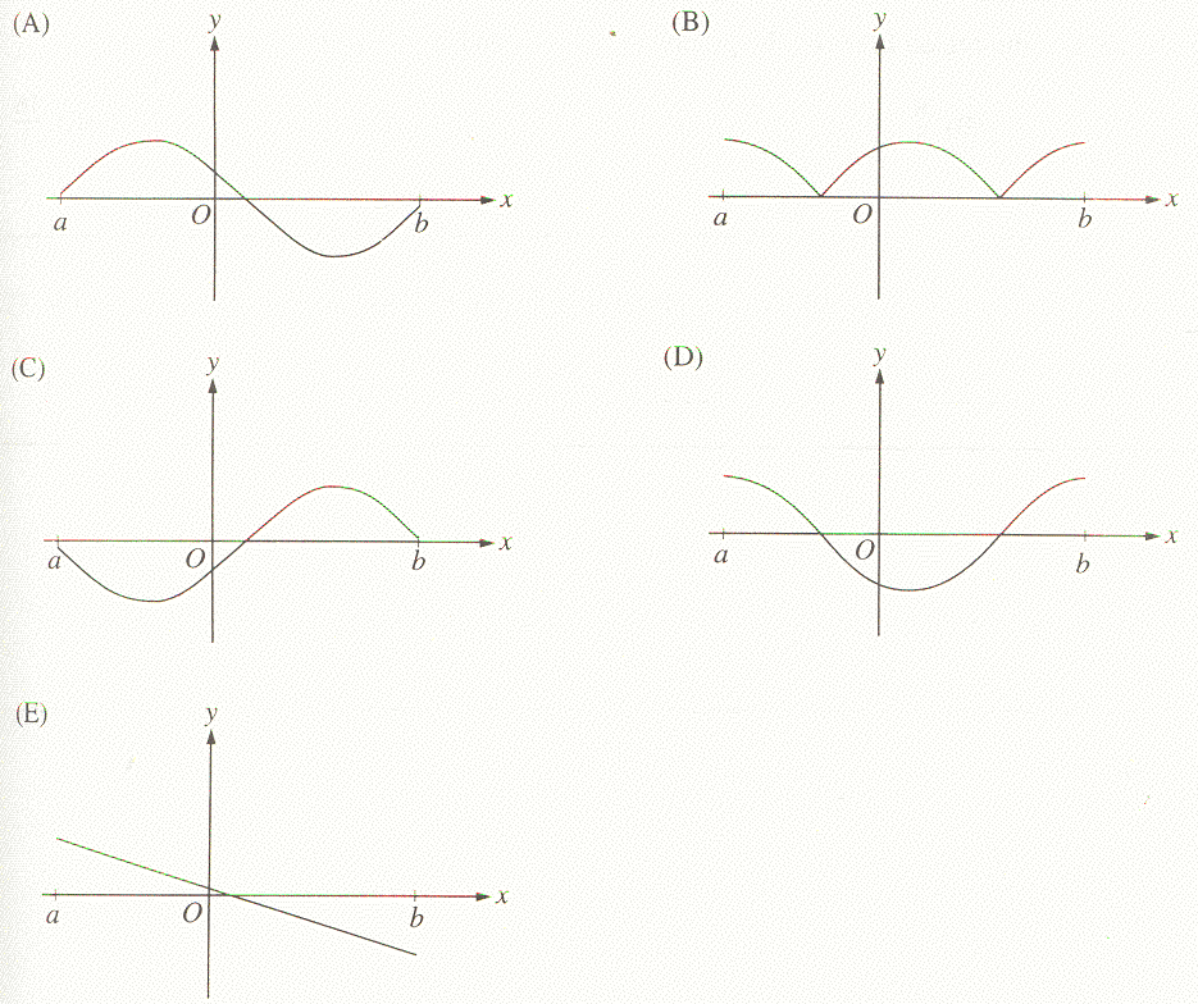
(C) $\frac{\sqrt{2}}{2}$

(D) 1

(E) $\sqrt{2}$



16. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?



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17. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2\cos 3$

(B) $-2\sin 3\cos 3$

(C) $6\cos 3$

(D) $2\sin 3\cos 3$

(E) $6\sin 3\cos 3$

18. A differentiable function f has the property that $f(5) = 3$ and $f'(5) = 4$. What is the estimate for $f(4.8)$ using local linear approximation for f at $x = 5$?

(A) 2.2

(B) 2.8

(C) 3.4

(D) 3.8

(E) 4.6

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19. The position of a particle moving along a line is given by $x(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $3 < t < 4$ only

(B) $t > 4$ only

(C) $t > 5$ only

(D) $0 < t < 3$ and $t > 5$

(E) $3 < t < 4$ and $t > 5$

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20. What is $\lim_{x \to \infty} \frac{x^3 - 9}{3 + 2x - x^2}$?

(A) -3

(B) $-\frac{1}{3}$

(C) -1

(D) 1

(E) The limit does not exist.

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21. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.

(B) The graph of the function has one point of inflection and the function has two relative extrema.

(C) The graph of the function has two points of inflection and the function has one relative extremum.

(D) The graph of the function has two points of inflection and the function has two relative extrema.

(E) The graph of the function has two points of inflection and the function has three relative extrema.

22. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval [-2, 4] occurs at x =

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

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23. If $f(x) = \sin^{-1} x$, then $f'\left(\frac{1}{2}\right) =$

(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{4}{5}$ (C) $-\frac{4}{5}$ (D) $-\frac{2\sqrt{3}}{3}$ (E) $\frac{\pi}{2}$

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24. A spherical balloon is inflated with gas at the rate of 800 cubic cm per minute. How fast is the radius of the balloon increasing at the instant the radius is 30 cm?
(HINT: $V = \frac{4}{3}\pi r^3$)

(A) $\frac{2}{9\pi}$ cm/min (B) $\frac{9}{2\pi}$ cm/min (C) 9 cm/min

(D) 2 cm/min (E) None of these

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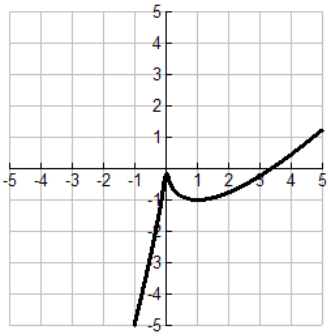
25. The graph shown represents $y = f(x)$. Which of the following is Not True?



(A) f is continuous on the interval [–1,1]

(B) $\lim_{x \to 0} f(x) = f(0)$

(C) f is concave up on $(0, \infty)$

(D) f has minimum at (–1, –5) and maximum at (1, –1) on the interval [–1,3]

(E) All are true

26. If f is continuous on $[-2,4]$ and $f(-2) = 5$, $f(0) = -3$, *and* $f(4) = 711$, then according to the Intermediate Value Theorem, how many zeroes are guaranteed on the closed interval $[-2, 4]$?

(A) none (B) one (C) two (D) three (E) four

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27. $\lim_{h\to 0} \dfrac{\cos\left(\pi/2 + h\right) - \cos\left(\pi/2\right)}{h} = ?$

(A) –1

(B) 0

(C) 1

(D) $\cos\left(\pi/2 + h\right)$

(E) undefined

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28. Let f and g be differentiable functions with the following properties:

I. $f(x) < 0$ for all x.

II. $g(5) = 2$

If $h(x) = \frac{f(x)}{g(x)}$ and $h'(x) = \frac{f'(x)}{g(x)}$, then $g(x) =$

(A) $\frac{1}{f'(x)}$

(B) $f(x)$

(C) $-f(x)$

(D) 0

(E) 2

PART II CALCULATOR: MULTIPLE-CHOICE
A calculator MAY be used on this part of the review

1. The first derivative of the function f is given by $f'(x) = \frac{\sin^2 x}{x} - \frac{2}{9}$. How many critical values does f have on the open interval (0,10)?

(A) One
(B) Two
(C) Three
(D) Four
(E) Six

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2. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$.
(B) $\lim_{x \to a} f(x)$ exists.
(C) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists.
(D) $f'(a)$ is defined.
(E) $f''(a)$ is defined.

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3. Let f be the function given by $f(x) = x^{2/3}$. Which of the following statements about f are true?

I. f is continuous at $x = 0$

II. f is differentiable at $x = 0$

III. f has an absolute minimum at $x = 0$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

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4. If $a \neq 0$, then $\lim_{x \to a} \frac{x^3 - a^3}{a^6 - x^6}$ is

(A) nonexistent
(B) 0
(C) $-\frac{1}{2a^3}$
(D) $-\frac{1}{a^3}$
(E) $\frac{1}{2a^3}$

5. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$. For $0 \le t \le 8$, g is decreasing most rapidly when t =

(A) 0.949 (B) 2.017 (C) 3.106 (D) 5.965 (E) 8.000

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6. If the length l of a rectangle is decreasing at a rate of 2 inches per minutes while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?

(A) A is always increasing.

(B) A is always decreasing.

(C) A is increasing only when $l > w$.

(D) A is increasing only when $l < w$.

(E) A remains constant.

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7. Which of the following is an equation of the line tangent to the graph of $f(x) = x^6 - x^4$ at the point where $f'(x) = -1$?

(A) $y = -x - 1.031$

(B) $y = -x - 0.836$

(C) $y = -x + 0.836$

(D) $y = -x + 0.934$

(E) $y = -x + 1.031$

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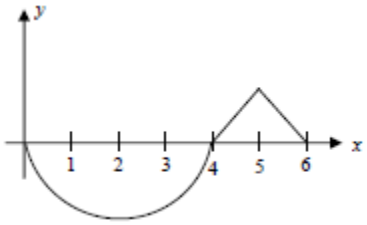
8. Let f be the function given by $f(x) = \tan x$ and let g be the function given by $g(x) = x^3$. At what value of x in the interval $0 \le x \le \pi$ do the graphs of f and g have parallel tangent lines?

(A) 0 (B) 0.75 (C) 1.883 (D) 1.697 (E) 10.63



9. The graph of f given above consists of two line segments and a semicircle. If $f(x) = g'(x)$, for what values of x is g increasing?

(A) (0,4) (B) (2,4) only (C) (2,5) (D) (4,5) only (E) (4,6)

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10. The graph of the function $y = x^5 - x^2 + \sin x$ has a point of inflection at x =

(A) 0.324 (B) 0.499 (C) 0.506 (D) 0.611 (E) 0.704

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11. Let h be the function defined by $h(x) = \cos 3x + \ln 4x$. What is the least value of x at which the graph of h changes concavity?

(A) 1.555 (B) 0.621 (C) 0.371 (D) 0.096 (E) 0.004

12. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - x - 12)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 4$

(B) f has a relative minimum at $x = -3$ and a relative maximum at $x = 4$

(C) f has a relative maximum at $x = 3$ and a relative minimum at $x = -4$

(D) f has a relative minimum at $x = 3$ and a relative maximum at $x = -4$

(E) It cannot be determined if f has any relative extrema.

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13. A particle moves on the x-axis with velocity given by $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \le t \le 3$. How many times does the particle change direction as t increases from –3 to 3?

(A) zero (B) one (C) two (D) three (E) four

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14. Let f be the function with the derivative given by $f'(x) = \cos(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three (D) Four (E) Five

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15. The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

(A) 2.545 meters (B) 10.263 meters (C) 34.125 meters (D) 54.889 meters (E) 89.005 meters

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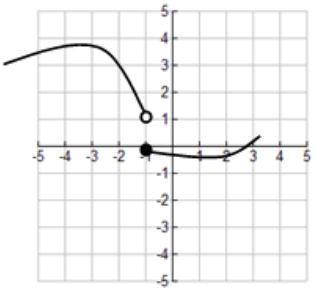

16. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $f(-1) = 1$ (B) $\lim_{x \to -1} f(x) = 0$ (C) $\lim_{x \to -1} f(x) = 1$

(D) $\lim_{x \to -1} f(x)$ does not exist (E) $f(-1)$ does not exist

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17. The function f has first derivative given by $f'(x) = \frac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f?

(A) 1.008 (B) 0.473 (C) 0

(D) –0.278 (E) The graph of f has no inflection points

PART III CALCULATOR: FREE-RESPONSE
A calculator MAY be used on this part of the review.

1. The function f is continuous on the closed interval [0, 10] and has values that are given in the table below. Using five equal subintervals, what is the left sum, right sum, midpoint sum approximations of $\int_0^{10} f(x)\,dx$?

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	–4.5	–12	–20.5	–30

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2. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$. Show that the number of mosquitoes is increasing at time $t = 6$. At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate?

PART IV NON-CALCULATOR: FREE-RESPONSE
NO calculator may be used on this part of the review.

1. The function $f(x) = x^3 + ax^2 + bx + c$ has a relative maximum at (–3, 25) and a point of inflection at $x = -1$. Find a, b, and c.

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2. Water is being pumped into a conical reservoir (vertex down) at the constant rate of 10π ft^3/min. If the reservoir has a radius of 4 ft and is 12 ft deep, how fast is the water rising when the water is 6 ft deep?

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3. State the set of values for which $f(x) = (x-2)(x-3)^2$ is BOTH increasing and concave up.

MULTIPLE-CHOICE ANSWER KEY

Part I: Non-Calculator

1. C
2. B
3. A
4. D
5. D
6. E
7. D
8. D
9. E
10. D
11. A
12. A
13. D
14. C
15. C
16. A
17. B
18. A
19. E
20. E
21. C
22. E
23. A
24. A
25. D
26. C
27. A
28. E

Part II: Calculator

1. B
2. E
3. E
4. C
5. B
6. C
7. A
8. C
9. E
10. B
11. B
12. B
13. C
14. C
15. B
16. D
17. B

FREE-RESPONSE ANSWER KEY

Part III: Calculator

1. Left sum = 2(20+18+12+2+–12) = 80
   Right Sum = 2(–30+–12+2+12+18) = –20
   Midpoint sum = 2(19.5+15.5+7.5+–4.5+–20.5) = 35

2. $R(6) = 4.438 > 0$ Since $R(6) > 0$, number of mosquitoes is increasing at $t = 6$.
   $R'(6) = -1.913 < 0$, $\therefore$ the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

Part IV: Non-Calculator

1. $a = 3$, $b = -9$, $c = -2$
2. $\frac{dh}{dt} = \frac{5}{2}$ ft/min. The water is rising at 5/2 ft/min when the water is 6ft deep.
3. f is both increasing and concave up on $(3, \infty)$