CALCULUS BC FINAL EXAM SEMESTER 1 REVIEW

PART I NON-CALCULATOR: MULTIPLE-CHOICE

NO calculator may be used on this part of the review.

1. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

(A) $2x - 3y = 0$

(B) $4x - 5y = 2$

(C) $4x - y = 10$

(D) $5x - 4y = 7$

(E) $5x - 2y = 13$

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2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

(A) $-\frac{3x+2y}{y^2}$

(B) $-\frac{3x+y}{x+y}$

(C) $\frac{1-3x-y}{x+y}$

(D) $-\frac{3x}{1+y}$

(E) $-\frac{3x}{x+y}$

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x	–1.0	–0.5	0.0	0.5	1.0	1.5	2.0
$g'(x)$	2	4	3	1	0	–3	–6

3. The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

(A) –6.5

(B) –1.5

(C) 1.5

(D) 2.5

(E) 3

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4. If $\frac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx} f(h(x)) =$

(A) $g(x^2)$

(B) $2xg(x)$

(C) $g'(x)$

(D) $2xg(x^2)$

(E) $x^2 g(x^2)$

5. If F' is a continuous function for all real x, then

$$\lim_{h\to 0} \frac{1}{h}\int_a^{a+h} F'(x)\,dx \text{ is}$$

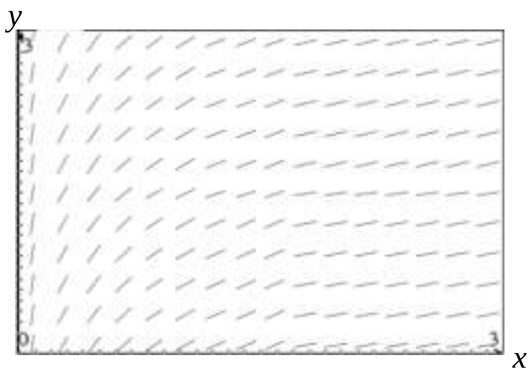
(A) 0

(B) $F(0)$

(C) $F(a)$

(D) $F'(0)$

(E) $F'(a)$



6. The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$

(B) $y = e^x$

(C) $y = e^{-x}$

(D) $y = \cos x$

(E) $y = \ln x$

7. $\int_0^3 \frac{dx}{(1-x)^2}$ is

(A) $-\frac{3}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

(E) None of these

8. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0,k]$, then $k =$

(A) 3

(B) $3^{1/2}$

(C) $18^{1/3}$

(D) $36^{1/4}$

(E) $36^{1/3}$

9. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

(C) $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^{x}$

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10. Determine the value of c so that $f(x)$ continuous on the entire real line when $f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$

(A) 0

(B) – 2

(C) 1

(D) $-\dfrac{1}{2}$

(E) None of these

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11. Find $\dfrac{dy}{dx}$ if: $x^2 + 3xy + y^3 = 10$

(A) $-\dfrac{2x+3y}{3x+3y^2}$

(B) $\dfrac{2x-3y}{3x+3y^2}$

(C) $-\dfrac{x+y}{x+y^2}$

(D) $\dfrac{x-y}{x+y^2}$

(E) None of these

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12. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top moving down the wall when the base of the ladder is 7 feet.

(A) $\frac{7}{12}$ ft/sec

(B) $-\frac{7}{12}$ ft/sec

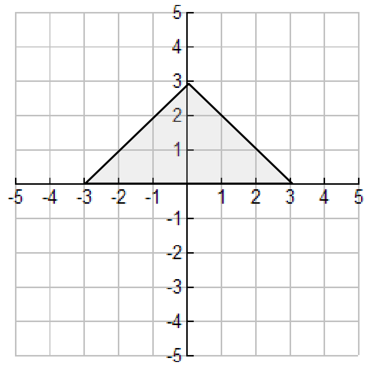
(C) $\frac{12}{7}$ ft/sec

(D) $-\frac{12}{7}$ ft/sec

(E) None of these

13. Set up a definite integral that yields the area of the region.

$$f(x) = 3 - |x|$$



(A) $\int_{3}^{3}(3-x)\,dx$

(B) $\int_{0}^{3}|x|\,dx$

(C) $\int_{3}^{-3}(3-|x|)dx$

(D) $\int_{-3}^{3}(3-|x|)dx$

(E) $\int_{3}^{0}|x|\,dx$

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14. A particle moves on a plane curve so that at any time $t > 0$, its position can be represented by: $x(t) = t^3 - \mathrm{t}$ and $y(t) = (2t-1)^3$. The acceleration vector of the particle at $t = 1$ is:

(A) (0, 1)

(B) (2, 3)

(C) (2, 6)

(D) (6, 12)

(E) (6, 24)

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15. If $f(x) = \sqrt{e^x}$, then $f'(\ln 2) =$

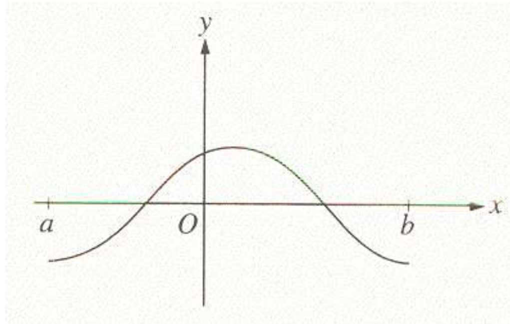
(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

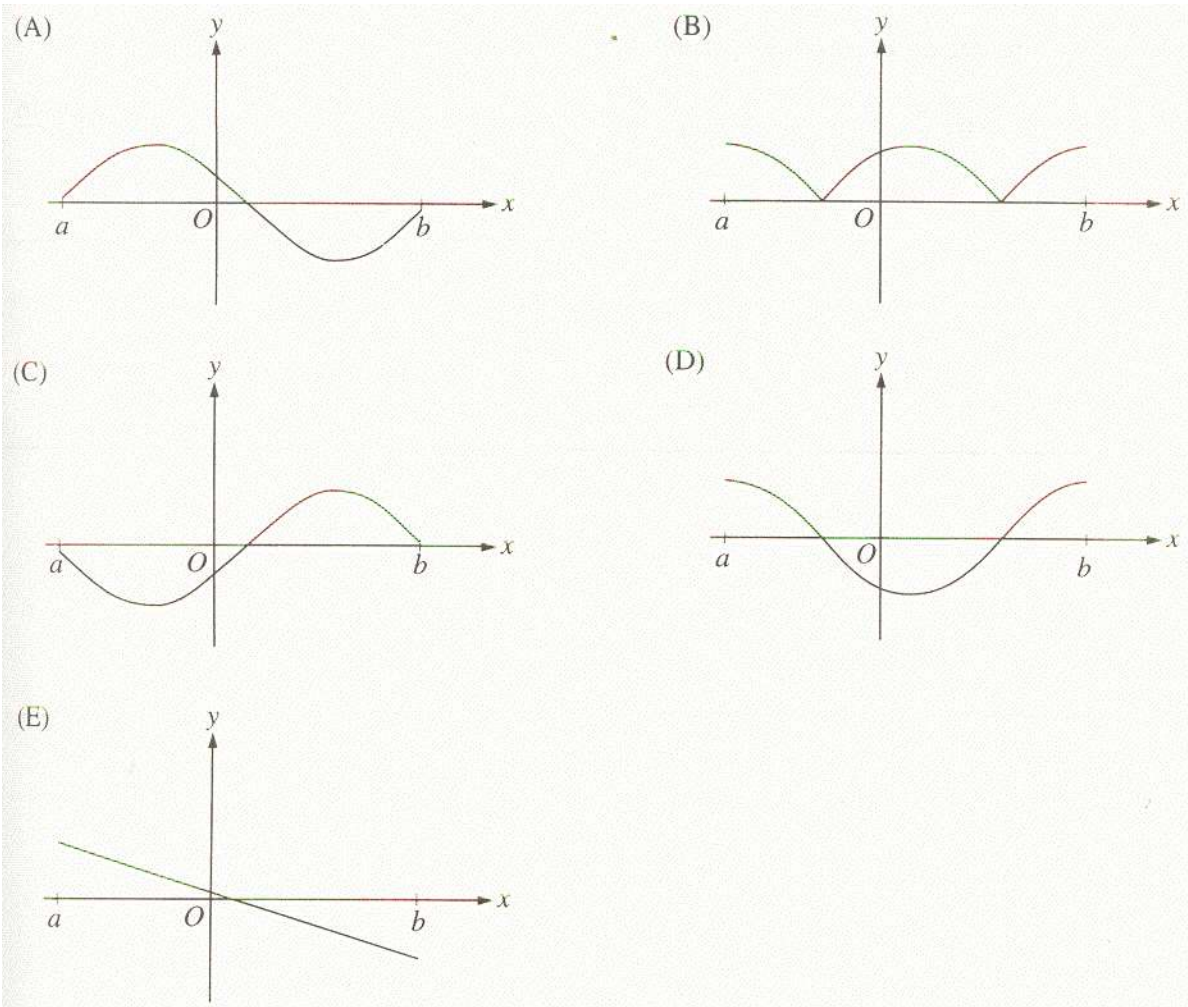
(C) $\frac{\sqrt{2}}{2}$

(D) 1

(E) $\sqrt{2}$



16. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?



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17. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0, 2]?

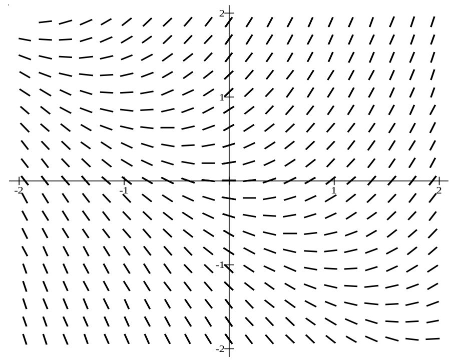
(A) $\frac{26}{9}$

(B) $\frac{52}{9}$

(C) $\frac{26}{3}$

(D) $\frac{52}{3}$

(E) 24



18. Shown above is a slope field for which of the following differential equations?

(A) $\dfrac{dy}{dx} = 1 + x$

(B) $\dfrac{dy}{dx} = x^2$

(C) $\dfrac{dy}{dx} = x + y$

(D) $\dfrac{dy}{dx} = \dfrac{x}{y}$

(E) $\dfrac{dy}{dx} = \ln y$

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19. If $F(x) = \int_0^{2x} \sqrt{t^3 + 1}\, dt$, then $F'(1) =$

(A) –3

(B) –2

(C) 3

(D) 6

(E) 18

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20. If $f(x) = x\sqrt{2x - 3}$, then $f'(x) =$

(A) $\dfrac{3x - 3}{\sqrt{2x - 3}}$ (B) $\dfrac{x}{\sqrt{2x - 3}}$ (C) $\dfrac{-x + 3}{\sqrt{2x - 3}}$

(D) $\dfrac{1}{\sqrt{2x - 3}}$ (E) $\dfrac{5x - 6}{\sqrt{2x - 3}}$

21. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval [-2, 4] occurs at x =

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

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22. Integrate: $\int xe^{x/2}\,dx$

(A) $2e^{x/2} + 4xe^{x/2} + C$ (B) $2xe^{x/2} + 4e^{x/2} + C$ (C) $\frac{1}{2}e^{x/2} + \frac{1}{4}xe^{x/2} + C$
(D) $2e^{x/2} + C$ (E) None of these

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23. If $f(x) = \sin^{-1} x$, then $f'\left(\frac{1}{2}\right) =$

(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{4}{5}$ (C) $-\frac{4}{5}$ (D) $-\frac{2\sqrt{3}}{3}$ (E) $\frac{\pi}{2}$

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24. A spherical balloon is inflated with gas at the rate of 800 cubic cm per minute. How fast is the radius of the balloon increasing at the instant the radius is 30 cm?
(HINT: $V = \frac{4}{3}\pi r^3$)

(A) $\frac{2}{9\pi}$ cm/min (B) $\frac{9}{2\pi}$ cm/min (C) 9 cm/min

(D) 2 cm/min (E) None of these

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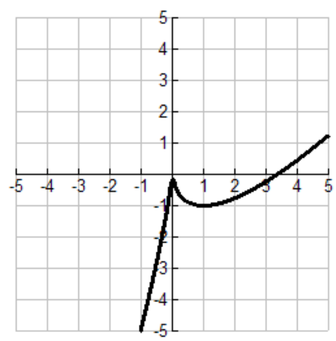
25. The graph shown represents $y = f(x)$. Which of the following is Not True?



(A) f is continuous on the interval [–1,1]

(B) $\lim_{x \to 0} f(x) = f(0)$

(C) f is concave up on $(0, \infty)$

(D) f has minimum at (–1, –5) and maximum at (1, –1) on the interval [–1,3]

(E) All are true

26. If f is continuous on $[-2,4]$ and $f(-2) = 5$, $f(0) = -3$, *and* $f(4) = 711$, then according to the Intermediate Value Theorem, how many zeroes are guaranteed on the closed interval $[-2, 4]$?

(A) none (B) one (C) two (D) three (E) four

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27. $\lim\limits_{h \to 0} \dfrac{\cos\left(\pi/2 + h\right) - \cos\left(\pi/2\right)}{h} = ?$

(A) -1

(B) 0

(C) 1

(D) $\cos\left(\pi/2 + h\right)$

(E) undefined

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28. $\int x\sqrt{3x}\,dx =$

(A) $\dfrac{2\sqrt{3}}{5}x^{5/2} + C$

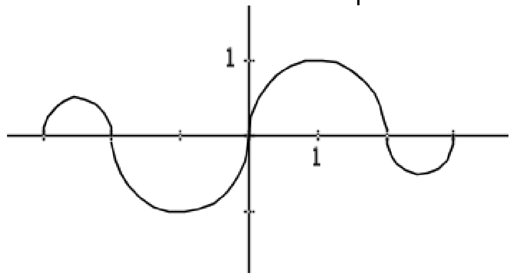
(B) $\dfrac{5\sqrt{3}}{2}x^{\frac{5}{2}} + C$

(C) $\dfrac{\sqrt{3}}{2}x^{\frac{1}{2}} + C$

(D) $2\sqrt{3x} + C$

(E) $\dfrac{5\sqrt{3}}{2}x^{\frac{3}{2}} + C$

PART II CALCULATOR: MULTIPLE-CHOICE
A calculator MAY be used on this part of the review



Graph of f

1. The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)dt$, where is $g(x)$ nonnegative?

(A) [–3, 3]

(B) [–3, 2] ∪ [0,2] only

(C) [0,3] only

(D) [0,2] only

(E) [–3, –2] ∪ [0,3] only

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2. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$.

(B) $\lim_{x\to a} f(x)$ exists.

(C) $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists.

(D) $f'(a)$ is defined.

(E) $f''(a)$ is defined.

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3. If the function f is defined by $f(x) = \sqrt{x^3+2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) –3.268

(B) –1.585

(C) 1.732

(D) 6.585

(E) 11.585

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4. Let g be the function given by $g(x) = \int_1^x 100(t^2 - 3t + 2)e^{-t^2}\,dt$. Which of the following statements about g must be true?

I. g is increasing on (1, 2).

II. g is increasing on (2, 3).

III. $g(3) > 0$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

5. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$. For $0 \le t \le 8$, g is decreasing most rapidly when t =

(A) 0.949

(B) 2.017

(C) 3.106

(D) 5.965

(E) 8.000

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6. What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$

(A) –3

(B) 0

(C) 3

(D) –3 and 3

(E) –3, 0, and 3

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7. If f is the function defined by $f(x) = \sqrt[3]{5x + x^2}$ and g is an antiderivative of f such that $g(5) = 8$, then $g(1) \approx$

(A) 3.375

(B) 2.665

(C) 1.817

(D) –3.375

(E) –2.665

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8. Let f be the function given by $f(x) = \tan x$ and let g be the function given by $g(x) = x^3$. At what value of x in the interval $0 \le x \le \pi$ do the graphs of f and g have parallel tangent lines?

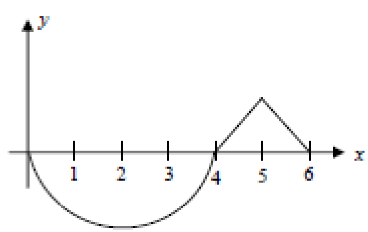
(A) 0

(B) 0.75

(C) 1.883

(D) 1.697

(E) 10.63



9. The graph of f given above consists of two line segments and a semicircle. If $g(x) = \int_4^x f(t)dt$, then $g(0) =$

(A) -4π (B) -2π (C) $-\pi$ (D) 2π (E) cannot be determined

10. The graph of the function $y = x^5 - x^2 + \sin x$ has a point of inflection at x =

(A) 0.324 (B) 0.499 (C) 0.506 (D) 0.611 (E) 0.704

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11. If $f(x) = 2g(x) - 1$ for $1 \leq x \leq 3$, then $\int_1^3 (f(x) + g(x))\, dx =$

(A) $\int_1^3 g(x)dx - 2$ (C) $2\int_1^3 g(x)dx - 2$ (E) $3\int_1^3 g(x)dx - x$

(B) $3\int_1^3 g(x)dx - 2$ (D) $2\int_1^3 g(x)dx - 1$

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12. Let h be the function defined by $h(x) = \cos 3x + \ln 4x$. What is the least value of x at which the graph of h changes concavity?

(A) 1.555 (B) 0.621 (C) 0.371 (D) 0.096 (E) 0.004

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13. A particle moves on the x-axis with velocity given by $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \leq t \leq 3$. How many times does the particle change direction as t increases from –3 to 3?

(A) zero (B) one (C) two (D) three (E) four

---

14. A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

(A) 2.909 (B) 3.062 (C) 6.884 (D) 9.016 (E) 47.393

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15. The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

(A) 2.545 meters (B) 10.263 meters (C) 34.125 meters (D) 54.889 meters (E) 89.005 meters

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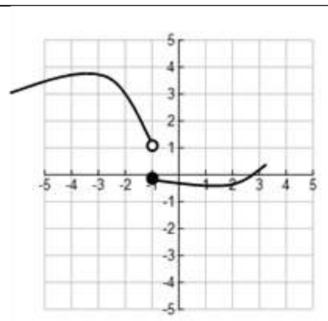

16. The graph of the function f is shown in the figure above. The value of $\lim_{x \to -1^+} \sin(f(x))$ is

(A) 0.909 (B) 0.540 (C) 0.017 (D) 0 (E) nonexistent

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17. Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects during the time interval $7 \leq t \leq 14$?

(A) 125 (B) 100 (C) 88 (D) 50 (E) 12

PART III CALCULATOR: FREE-RESPONSE

A calculator MAY be used on this part of the review.

1. The function f is continuous on the closed interval [0, 10] and has values that are given in the table below. Using five equal subintervals, what is the left sum, right sum, midpoint sum, and trapezoidal approximations of $\int_0^{10} f(x)\,dx$?

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	-4.5	-12	-20.5	-30

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2. If $\frac{dy}{dx} = \frac{x}{\sqrt{9+x^2}}$ and $y = 5$, when $x = 4$, find the equation of the curve.

PART IV NON-CALCULATOR: FREE-RESPONSE

NO calculator may be used on this part of the review.

1. The function $f(x) = x^3 + ax^2 + bx + c$ has a relative maximum at (–3, 25) and a point of inflection at $x = -1$. Find a, b, and c.

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2. Water is being pumped into a conical reservoir (vertex down) at the constant rate of 10π ft^3/min. If the reservoir has a radius of 4 ft and is 12 ft deep, how fast is the water rising when the water is 6 ft deep?

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3. For what values of t does the curve given by the parametric equations

$$x(t) = t^3 - t^2 - 1$$
$$y(t) = t^4 + 2t^2 - 8t$$

have a vertical tangent?

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4. State the set of values for which $f(x) = (x-2)(x-3)^2$ is BOTH increasing and concave up.

MULTIPLE-CHOICE ANSWER KEY

Part I: Non-Calculator

1. D
2. B
3. D
4. D
5. E
6. E
7. A
8. E
9. C
10. D
11. A
12. A
13. D
14. E
15. C
16. A
17. A
18. C
19. D
20. A
21. E
22. B
23. A
24. A
25. D
26. C
27. A
28. A

Part II: Calculator

1. A
2. E
3. B
4. B
5. B
6. A
7. D
8. C
9. B
10. B
11. B
12. B
13. C
14. C
15. B
16. D
17. A

FREE-RESPONSE ANSWER KEY

Part III: Calculator

1. Left sum = 2(20+18+12+2+–12) = 80
   Right Sum = 2(–30+–12+2+12+18) = –20
   Midpoint sum = 2(19.5+15.5+7.5+–4.5+–20.5) = 35
   Trapezoid sum = ½ (2)[20+2(18)+2(12)+2(2)+2(–12)+ –30] = 30

2. $y = \frac{1}{2}\ln(9 + x^2) + 5 - \ln 5$

Part IV: Non-Calculator

1. $a = 3$, $b = -9$, $c = -2$
2. $\frac{dh}{dt} = \frac{5}{2}$ ft/min. The water is rising at 5/2 ft/min when the water is 6ft deep.
3. $t = 0$, $t = 2/3$
4. f is both increasing and concave up on $(3, \infty)$