- Vectors
- Partial Fractions
- Polar
- Parametric

Limits

Analytically

ex: $$\lim_{x\to 4} \frac{x^2+3x-28}{x-4}$$

$$= \lim_{x\to 4} \frac{(x+7)\cancel{(x-4)}}{\cancel{x-4}}$$

$$= 11$$

ex: $$\lim_{x\to 0} \frac{\sin 5x}{x}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$= \lim_{x\to 0} 5\cdot \underbrace{\boxed{\frac{\sin 5x}{5x}}}_{1}$$

$$= 5\cdot 1$$

$$= 5$$

ex: $f(x) = \begin{cases} 3x+2 & x \geq 1 \\ 4 & x < 1 \end{cases}$

$\lim_{x \to 1} f(x)$

right

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3x+2) = 5$

left

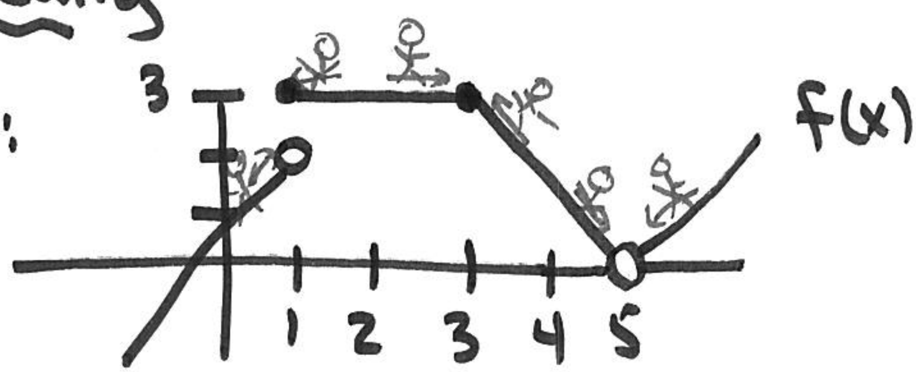
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} 4 = 4$

$\left.\begin{matrix} 5 \\ 4 \end{matrix}\right\} \neq$

so, $\lim_{x \to 1} f(x)$ DNE

Graphically

ex:



$\lim_{x \to 3} f(x) = 3$

$\lim_{x \to 1} f(x) = \text{DNE}$ b/c $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

$\lim_{x \to 5} f(x) = 0$

Numerically (table of values)

ex: $\lim_{x \to 4} \frac{x^2+3x-28}{x-4} = 11$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
f(x)	10.9	10.99	10.999		11.001	11.01	11.1

Infinite Limits & Limits @ Infinity (End Behavior)

$\lim_{x \to 2^+} \frac{1}{x-2} = \infty$

$\frac{1}{2.001-2} = \frac{1}{.001} = \frac{1}{1/1000} = 1000$

$\lim_{x \to 2^-} \frac{1}{x-2} = -\infty$

$\frac{1}{1.99-2} = \frac{1}{-.01} = -100$

$\lim_{x \to \infty} \frac{2x^2-3}{5x^2+1} = \frac{2}{5}$

exp N = exp D
coefficients

$\lim_{x \to \infty} \frac{2x^2-3}{5x^3+1} = 0$

exp N < exp D
$y = 0$

$\lim_{x \to \infty} \frac{2x^3-3}{5x+1} = \infty$

exp N > exp D
$\pm\infty$

Chapter 2 Topics

- Parametric
- Polar
- Partial Fractions
- Vectors
- Limits (Analytically, Numerically, Graphically)
- Infinite Limits
- Limits @ Infinity
- Continuity

A function is continuous @ $x = c$

① $f(c)$ exist (y-value)

② $\lim\limits_{x \to c} f(x)$ (check $\lim\limits_{x \to c^-} f(x) = \lim\limits_{x \to c^+} f(x)$)

③ $f(c) = \lim\limits_{x \to c} f(x)$

ex: $\cancel{\lim\limits_{x \to 4}}\ f(x)\ \dfrac{x^2 + 3x - 28}{x - 4}$ @ $x = 4$

continuous?

$f(4) = $ DNE , $f(x)$ discont @ $x = 4$

$$f(x) = \frac{(x+7)\cancel{(x-4)}}{\cancel{x-4}}$$

→ removable discontinuity @ $x=4$ (HOLE)

ex: $f(x) = \begin{cases} 3x+2 & x \geq 1 \\ 5 & x < 1 \end{cases}$

Is $f(x)$ continuous?

① $f(1) = 3(1)+2 = 5$ ✓

② $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} 5 = 5$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (3x+2) = 5$

$=$, $\lim\limits_{x\to 1} f(x) = 5$

③ $f(1) = \lim\limits_{x\to 1} f(x)$

So, $f(x)$ is continuous @ $x=1$

Discontinuities

Removable

* HOLE

$\lim\limits_{x\to c^-} f(x) = \lim\limits_{x\to c^+} f(x)$

but $\lim\limits_{x\to c} f(x) \neq f(c)$

Non-Removable

* JUMP

$\lim\limits_{x\to c^-} f(x) \neq \lim\limits_{x\to c^+} f(x)$

* Infinite (Asymptotes)

$$\lim_{x \to c^-} f(x) = \pm\infty$$

or $$\lim_{x \to c^+} f(x) = \pm\infty$$

<u>Average</u> <u>Rate</u> <u>of</u> <u>Change</u>

Slope $\quad \dfrac{\Delta y}{\Delta x} \quad \dfrac{y_2 - y_1}{x_2 - x_1}$

ex: $f(x) = x^2 + 3x - 2$ on $[\overset{a}{1}, \overset{b}{2}]$

$$\text{avg rate of change} = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{f(2) - f(1)}{2 - 1}$$

$$= \frac{8 - 2}{1}$$

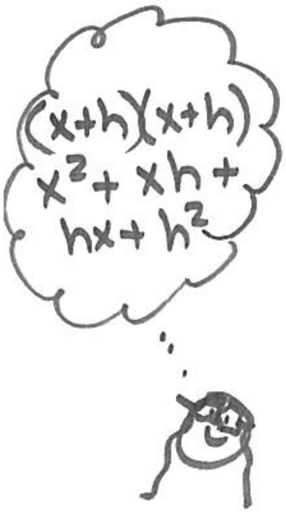
$$= 6$$

<u>Instantaneous</u> <u>Rate</u> <u>of</u> <u>Change</u> (Slope of Tangent Line)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

ex: $f(x) = x^2 + 3x - 2$ @ $x = 1$

$$\lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - 2 - (x^2+3x-2)}{h}$$



$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h - \cancel{2} - \cancel{x^2} - \cancel{3x} + \cancel{2}}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x+h+3)}{\cancel{h}}$$

$$\lim_{h \to 0} (2x+h+3)$$

$$f'(x) = 2x+3$$

"f prime of x"

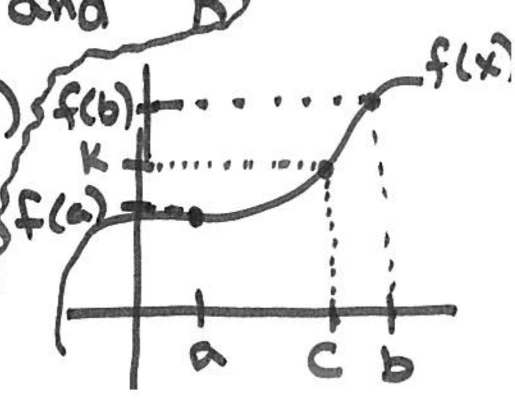
$$f'(1) = 2(1)+3$$
$$= 5$$

<u>Intermediate</u> <u>Value</u> <u>Theorem</u> (IVT)

If $f(x)$ is cont. on $[a,b]$ and k is between $f(a)$ and $f(b)$ then $\exists$ some #, c, on $[a,b]$ s.t. $f(c) = k$

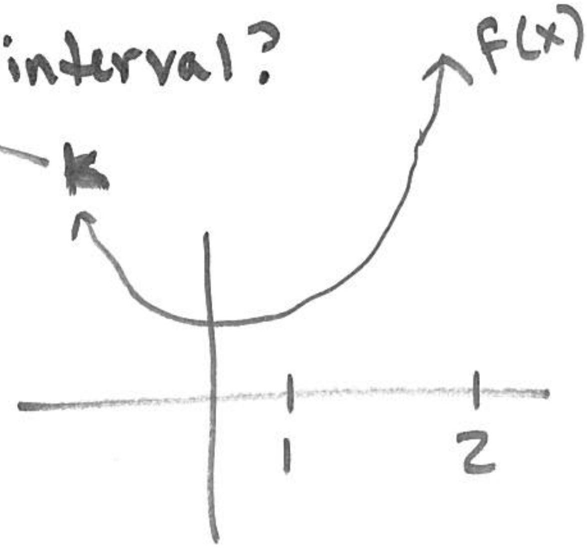
there exists

such that



ex: $f(x) = 3x^2+2$ on $[\overset{a}{1}, \overset{b}{2}]$

Will $f(x)=0$ on that interval?
(Will IVT work?)



f cont? yes b/c
f is polynomial

$f(1) = 3(1)^2+2 = 5$
$f(2) = 3(2)^2+2 = 14$

Since $f(1) > 0$ and $f(2) > 0$,
then $f(x) \neq 0$ on that interval.