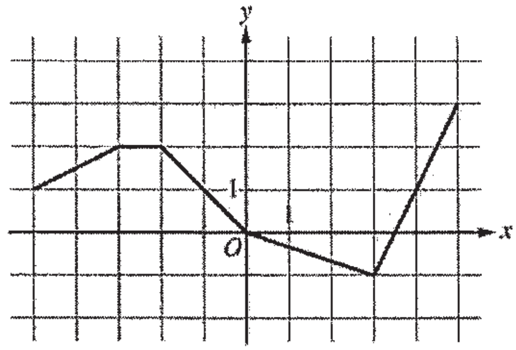
Finding the Errors in Student Work

Given the following AP Free-Response Question, view the students' work and determine where the students made any errors, if any.

2017 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

x	$g(x)$	$g'(x)$
−5	10	−3
−4	5	−1
−3	2	4
−2	3	1
−1	1	−2
0	0	−3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

Work for Student 1

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

$f(x) = \cos(2x) + e^{\sin x}$

$f'(x) = -2\sin(2x) + (\cos x)e^{\sin x}$

$f'(\pi) = -2\sin(2\pi) + (\cos\pi)e^{\sin\pi}$

$f'(\pi) = -2(0) + (-1)e^{(0)}$

$f'(\pi) = 0 + -1(1) = -1$

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

$k(x) = h(f(x))$

$k'(x) = h'(f(x))\,f'(x)$

$k'(\pi) = h'(f(\pi))\,f'(x)$

$k'(\pi) = [h'(2)](-1)$

$k'(\pi) = (-\frac{1}{3})(-1)$

$f(\pi) = \cos(2\pi) + e^{\sin\pi}$

$f(\pi) = 1 + e^{0} = 1 + 1 = 2$

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

$m(x) = g(-2x) \cdot h(x)$

$m'(x) = -2g'(-2x) \cdot h(x) + h'(x) \cdot g(-2x)$

$m'(2) = -2g'(-4) \cdot h(2) + h'(2) \cdot g(-4)$

$m'(2) = -2[-1] \cdot \frac{2}{3} + (\frac{1}{3}) \cdot 5$

$m'(2) = \frac{8}{3} + \frac{5}{3} = \frac{13}{3}$

Work for Student 2

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

$f'(x) = -2\sin(2x) + e^{\sin x}(\cos(x))$

$f'(\pi) = -2\sin(2\pi) + e^{\sin \pi} \cos(\pi)$

$0 + 1(-1)$

$\boxed{f'(\pi) = -1}$

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

$k'(x) = h'(f(x))f'(x)$

$k'(\pi) = h'(f(\pi))f'(\pi)$

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

$m'(x) = g'(-2x)(-2) \cdot h'(x)$

$m'(2) = g'(-4) -2 \cdot h'(-2)$

$m'(2) = (4 - 2) \cdot -1$

$m'(2) = 2 \cdot -1$

$\boxed{m'(2) = -2}$

Work for Student 3

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

$f'(x) = -2\sin(2x) + e^{\sin x} \frac{1}{\cos(x)}$

$f'(\pi) = -2\sin(2\pi) + e^{\sin \pi} \frac{1}{\cos(\pi)}$

$-2\sin(0) + e^{\sin 0}$

$= -1$

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

$k'(x) = h'(f(x))f'(x)$

$k'(\pi) = h'(f(\pi)) \cdot f'(\pi)$

$= h'(\cos 2\pi + e^0 \quad) \cdot -1$

$= -h'(1) = +1/3$

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

$m'(x) = g'(-2x) \cdot -2 \cdot h(x) - g(-2x) \cdot h'(x)$

$m'(2) = g'(-4) \cdot -2 \cdot h(2) - g(-4) \cdot h'(2)$

$= -1 \cdot -2 \cdot \frac{2}{3} - 5 \cdot -1/3 = -\frac{-11}{3} + 5/3$

$= \frac{-6}{3} = -2$

AP Multiple-Choice Exponential & Logarithmic Functions

4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

18. $\lim_{h\to 0} \frac{\ln(4+h) - \ln(4)}{h}$ is

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2x$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{\left(-2/x^2\right)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

Products, Quotients, and Composite Functions

a) For each of the functions $f(x)$ given below, decide whether the function is a product, a quotient, or a composition of simpler functions, and then complete the second and third columns of the table.

Example: $f(x) = x^3 \sec x$ Check one: ☑ Product: $u(x) \cdot v(x)$ ☐ Quotient: $\frac{u(x)}{v(x)}$ ☐ Composition: $u(v(x))$	$u(x) = x^3$	$v(x) = \sec x$
1. $f(x) = \cos(\ln x)$ Check one: ☐ Product: $u(x) \cdot v(x)$ ☐ Quotient: $\frac{u(x)}{v(x)}$ ☐ Composition: $u(v(x))$	$u(x) =$	$v(x) =$
2. $f(x) = \tan^{-1}\left(\sqrt{x}\right)$ Check one: ☐ Product: $u(x) \cdot v(x)$ ☐ Quotient: $\frac{u(x)}{v(x)}$ ☐ Composition: $u(v(x))$	$u(x) =$	$v(x) =$
3. $f(x) = \frac{\sin x}{1 + \sin x}$ Check one: ☐ Product: $u(x) \cdot v(x)$ ☐ Quotient: $\frac{u(x)}{v(x)}$ ☐ Composition: $u(v(x))$	$u(x) =$	$v(x) =$
4. $f(x) = e^x \csc x$ Check one: ☐ Product: $u(x) \cdot v(x)$ ☐ Quotient: $\frac{u(x)}{v(x)}$ ☐ Composition: $u(v(x))$	$u(x) =$	$v(x) =$

5. $f(x)=e^{\sin^{-1}x}$ Check one: ☐ Product: $u(x)\cdot v(x)$ ☐ Quotient: $\frac{u(x)}{v(x)}$ ☐ Composition: $u(v(x))$	$u(x)=$	$v(x)=$
6. $f(x)=\frac{3^x}{3^x+x}$ Check one: ☐ Product: $u(x)\cdot v(x)$ ☐ Quotient: $\frac{u(x)}{v(x)}$ ☐ Composition: $u(v(x))$	$u(x)=$	$v(x)=$
7. $f(x)=\sqrt[3]{x}\ln x$ Check one: ☐ Product: $u(x)\cdot v(x)$ ☐ Quotient: $\frac{u(x)}{v(x)}$ ☐ Composition: $u(v(x))$	$u(x)=$	$v(x)=$

b) Find $f'(x)$ for problems #1, 2, 5, and 7.

NO CALCULATOR ALLOWED

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

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Continue problem 6 on page 21.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

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GO ON TO THE NEXT PAGE.

AP Multiple-Choice Inverse Functions & Inverse Trigonometric Functions

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

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27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

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20. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

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28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.