1

$l = 10\,ft$

$\frac{dx}{dt} = 2\,ft/sec$

$y = 6\,ft$

$x = 8\,ft$

$\frac{dy}{dt} = ?$

(l is constant)

$x^2 + y^2 = l^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(8)(2) + 2(6)\frac{dy}{dt} = 0$

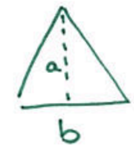
$32 + 12\frac{dy}{dt} = 0$

$12\frac{dy}{dt} = -32$

$\frac{dy}{dt} = -8/3\ ft/sec$

The top of ladder is moving down the wall at a rate of 8/3 ft/sec

2



$\frac{da}{dt} = 1\,cm/min$

$\frac{dA}{dt} = 2\,cm^2/min$

$\frac{db}{dt} = ?$

$a = 10\,cm$

$A = 100\,cm^2$

$A = \frac{1}{2}ba$

$100 = \frac{1}{2}b(10)$

$100 = 5b$

$20 = b$

$A = \frac{1}{2}ba$

$\frac{dA}{dt} = \frac{1}{2}\left(a\cdot\frac{db}{dt} + b\cdot\frac{da}{dt}\right)$

$2 = \frac{1}{2}\left(10\cdot\frac{db}{dt} + 20\cdot 1\right)$

$2 = \frac{1}{2}\left(10\frac{db}{dt} + 20\right)$

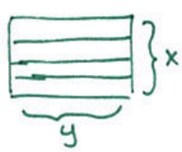
$2 = 5\frac{db}{dt} + 10$

$-8 = 5\frac{db}{dt}$

$\frac{db}{dt} = -8/5\ cm/min$

The base is changing at a rate of $-8/5$ cm/min

3



$P = 750\,ft$

max Area = ?

$P = 2x + 5y$

$750 = 2x + 5y$

$750 - 2x = 5y$

$150 - \frac{2}{5}x = y$

$A = xy$

$A = x(150 - \frac{2}{5}x)$

$A = 150x - \frac{2}{5}x^2$

$A'(x) = 150 - \frac{4}{5}x$

$0 = 150 - \frac{4}{5}x$

$\frac{4}{5}x = 150$

$x = \frac{375}{2}$

$y = 150 - \frac{2}{5}\left(\frac{375}{2}\right)$

$y = 75$

$A''(x) = -\frac{4}{5}$

$A''\left(\frac{375}{2}\right) = -\frac{4}{5} < 0$

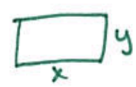
$\therefore$, $x = \frac{375}{2}$ is rel. max

Area $= xy$

$= \left(\frac{375}{2}\right)(75)$

$= \boxed{14062.5\,ft^2}$

4



$A = 1000\,m^2$

min P = ?

$A = xy$

$1000 = xy$

$x = \frac{1000}{y}$

$P = 2x + 2y$

$P = 2\left(\frac{1000}{y}\right) + 2y$

$P = 2000y^{-1} + 2y$

$P'(y) = -2000y^{-2} + 2$

$0 = -\frac{2000}{y^2} + 2$

$\frac{2000}{y^2} = 2$

$2000 = 2y^2$

$1000 = y^2$

$\sqrt{1000} = y$

$10\sqrt{10} = y$

$x = \frac{1000}{10\sqrt{10}}$

$x = \frac{100}{\sqrt{10}}$

or $10\sqrt{10}$

$P''(y) = 4000y^{-3}$

$P''(10\sqrt{10}) = 4000(10\sqrt{10})^{-3} > 0$

$\therefore$, $y = 10\sqrt{10}$ is a rel. min

Dimensions of □ are: $10\sqrt{10} \times 10\sqrt{10}$

(5)

crit # when $f'(x) = 0$ or DNE

check crit #s + endpts

$f(0) = 0$

$f(1) = 1$

$f(-3) = 1.552$

abs max @ $(-3, 1.552)$ b/c highest y-value from crit # + endpts

abs min @ $(0,0)$ b/c lowest y-value from crit #s + endpts

$\boxed{\text{abs max } 1.552, \text{ abs min } 0}$

(6)

MVT exists b/c $f(x)$ is diff'able + cont on $[-3, -2/3]$.

($f(x)$ is not diff'able @ $x=0$)

$f(x) = -\frac{1}{x} = -x^{-1}$

$f'(x) = x^{-2}$

$\frac{f(b)-f(a)}{b-a}$ slope of secant line

$= \frac{f(-2/3) - f(-3)}{-2/3 - (-3)}$

$= \frac{3/2 - 1/3}{-2/3 + 3}$

$= \frac{1}{2}$

$\frac{1}{x^2} = \frac{1}{2}$

$x^2 = 2$

$x = \pm\sqrt{2}$ only $-\sqrt{2}$ is in interval

$\boxed{-\sqrt{2}}$

(7)

f dec on $(-5/6, 7/2)$ b/c $f' < 0$ on $(-5/6, 7/2)$

f inc on $(-\infty, -5/6) \cup (7/2, \infty)$ b/c $f' > 0$ on those intervals

f has inf. pt @ $(4/3, 9.370)$ b/c f'' changes signs @ $x = 4/3$

f is concave up on $(4/3, \infty)$ b/c $f'' > 0$ on $(4/3, \infty)$

f is concave down on $(-\infty, 4/3)$ b/c $f'' < 0$ on $(-\infty, 4/3)$

(8)

$f'(x) = 12x^2 - 32x - 35$

$0 = 12x^2 - 32x - 35$

$x = 7/2, \quad x = -5/6$

$f''(x) = 24x - 32$

$f''(7/2) > 0$, $\therefore$ rel min @ $(7/2, -72)$

$f''(-5/6) < 0$, $\therefore$ rel. max @ $(-5/6, 90.741)$

(9)

$f(x) = x^{2/3}$ $\quad a = 8, \ x = 8.1$

$f(8) = 8^{2/3}$

$= (\sqrt[3]{8})^2 = 4$

$f'(x) = \frac{2}{3} x^{-1/3}$

$f'(8) = \frac{2}{3}(8)^{-1/3}$

$= \frac{2}{3} \cdot \frac{1}{\sqrt[3]{8}}$

$= \frac{2}{3} \cdot \frac{1}{2}$

$= \frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{3}(x - 8)$

$y - 4 = \frac{1}{3}(8.1 - 8)$

$y - 4 = \frac{1}{3}(.1)$

$y - 4 = .033$

$y = 4.033$

approx value of $8.1^{2/3}$ is $\boxed{4.033}$

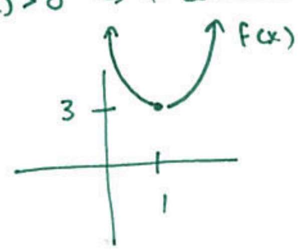
(10)

$f(1) = 3 \rightarrow$ pt $(1, 3)$

$f'(x) > 0$ when $x > 1 \rightarrow f$ inc on $(1, \infty)$

$f'(x) < 0$ when $x < 1 \rightarrow f$ dec on $(-\infty, 1)$

$f''(x) > 0 \rightarrow f$ concave up on $(-\infty, \infty)$

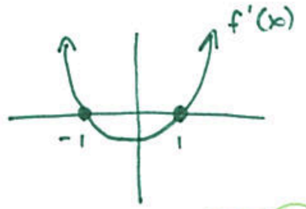


(11)

a) $f'(x)=0$ @ $x=-1, 1$ b/c $f(x)$ has horizontal tangent lines @ $x=-1, 1$

$f'(x)>0$ on $(-\infty,-1)\cup(1,\infty)$ b/c $f(x)$ inc on those intervals

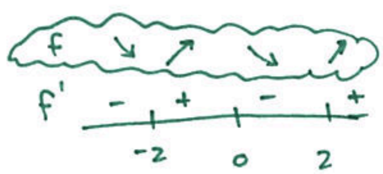
$f'(x)<0$ on $(-1,1)$ b/c $f(x)$ dec on $(-1,1)$



b) $f'(x)=0$ @ $x=-2, 0, 2$ $\therefore$, $f(x)$ has horizontal tangent lines @ $x=-2, 0, 2$

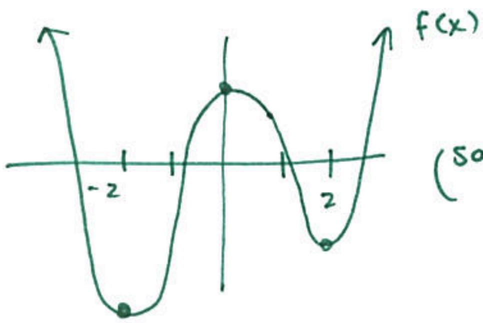
$f'(x)>0$ on $(-2,0)\cup(2,\infty)$ $\therefore$, $f(x)$ is increasing on those intervals

$f'(x)<0$ on $(-\infty,-2)\cup(0,2)$ $\therefore$, $f(x)$ is dec on those intervals



f has rel. min @ $x=-2$ and $x=2$ b/c f' changes from neg to pos @ $x=-2$ and $x=2$

f has rel. max @ $x=0$ b/c f' changes from pos to neg @ $x=0$



(sample picture)