

①  $\int f(x)e^x dx = f(x)e^x - \int 4x^3 e^x dx$

$u = f(x) \quad dv = e^x dx$   
 $du = f'(x) dx \quad v = e^x$

$\int f(x)e^x dx = f(x)e^x - \int e^x \cdot f'(x) dx$

$f'(x) = 4x^3$   
 $f(x) = 4\left(\frac{1}{4}x^4\right) + C$   
 $F(x) = x^4$

②  $\int_0^1 f(x)g'(x) dx \quad u=f(x) \quad dv=g'(x)$

$du=f'(x)dx \quad v=g(x)$

$= f(x) \cdot g(x) \Big|_0^1 - \int_0^1 g(x) \cdot f'(x) dx$

$= f(1)g(1) - f(0)g(0) - \int_0^1 f'(x)g(x) dx$

$= 3(-1) - 6(1) - 4$

$= \boxed{-13}$

③  $f'(x) = x^2 \ln x$

$\int f'(x) dx = \int x^2 \ln x dx$

$f(x) = \ln x \cdot \frac{1}{3}x^3 - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx$

$f(x) = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

$f(e) = \frac{1}{3}e^3 \ln e - \frac{1}{9}e^3 + C$

$2 = \frac{1}{3}e^3 - \frac{1}{9}e^3 + C$

$2 = \frac{2}{9}e^3 + C \rightarrow C = 2 - \frac{2}{9}e^3$

$u = \ln x \quad dv = x^2 dx$   
 $du = \frac{1}{x} dx \quad v = \frac{1}{3}x^3$

$\therefore f(x) = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + 2 - \frac{2}{9}e^3$

④ avg # of people in line

$= \frac{1}{5-0} \int_0^5 L(t) dt$

$= \frac{1}{5} \int_0^5 L(t) dt$

$= \boxed{151.667 \text{ people}}$

⑤  $\frac{d}{dx} \int_0^{x^2} (\cos(t^3)) dt$

$= \cos(x^2)^3 \cdot 2x$

$= 2x \cos x^6$

⑥  $f'(x) = \frac{10x}{x^2+x-6}$

$\int f'(x) dx = \int \frac{10x}{x^2+x-6} dx$

$f(x) = \int \left( \frac{6}{x+3} + \frac{4}{x-2} \right) dx$

$= 6 \ln|x+3| + 4 \ln|x-2| + C$

$= \ln(x+3)^6 + \ln(x-2)^4 + C$

$F(x) = \ln[(x+3)^6(x-2)^4] + C$

$\frac{10x}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$

$10x = A(x-2) + B(x+3)$

$10x = Ax - 2A + Bx + 3B$

$A+B=10 \quad -2A+3B=0$

$A+4=10$

$A=6$

$\frac{2A+2B=20}{5B=20}$

$B=4$

$B=4$