Some Practice Problems for Some Concepts in Unit 6

1. Using Euler's Method, approximate the particular solution to $y(0.3)$ of the differential equation $\frac{dy}{dx} = x + e^{-y}$ passing through $(0,1)$ using 3 increments of equal size.

$\Delta x = \frac{0.3-0}{3} = 0.1$

(x,y)	$\frac{dy}{dx}$	$\frac{dy}{dx}\Delta x$	$\frac{dy}{dx}\Delta x + y_1$
$(0,1)$	e^{-1}	$e^{-1}(.1)$	$.1e^{-1}+1 \approx 0.037$
$(.1, 0.037)$	1.137	0.114	$.114 + .037 = 0.151$
$(.2, 0.151)$	1.262	0.126	$0.126 + 0.151 = 0.276$

$y(0.3) \approx 0.276$

2. Solve the differential equation $\frac{dy}{dx} = \frac{x}{y}\sin x$ for $y(0) = 4$.

$\frac{dy}{dx} = \frac{x\sin x}{y}$

$\int y\,dy = \int x\sin x\,dx$

tabular by parts

u	dv
x	$+\ \sin x$
1	$-\ -\cos x$
0	$-\sin x$

$\frac{1}{2}y^2 = -x\cos x + \sin x + C$

$\frac{1}{2}(0)^2 = -4\cos 4 + \sin 4 + C$

$4\cos 4 - \sin 4 = C$

$\frac{1}{2}y^2 = -x\cos x + \sin x + 4\cos 4 - \sin 4$

$y^2 = -2x\cos x + 2\sin x + 8\cos 4 - 2\sin 4$

$y = \pm\sqrt{-2x\cos x + 2\sin x + 8\cos 4 - 2\sin 4}$

positive b/c $y(0) > 0$

$y = \sqrt{-2x\cos x + 2\sin x + 8\cos 4 - 2\sin 4}$

3. Find and sketch the particular solution to $\frac{dy}{dx} = \frac{\ln x}{x}$ through the point $(1, -2)$.

$\frac{dy}{dx} = \frac{\ln x}{x}$

$\int dy = \int \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$y = \int u\,du$

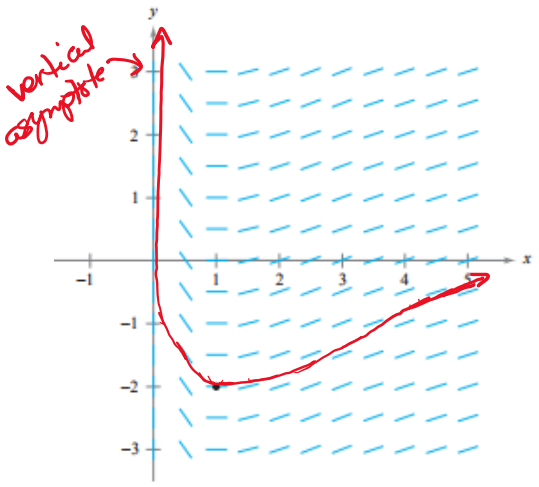
$y = \frac{1}{2}u^2 + C$

$y = \frac{1}{2}(\ln x)^2 + C$

$-2 = \frac{1}{2}(\ln 1)^2 + C$

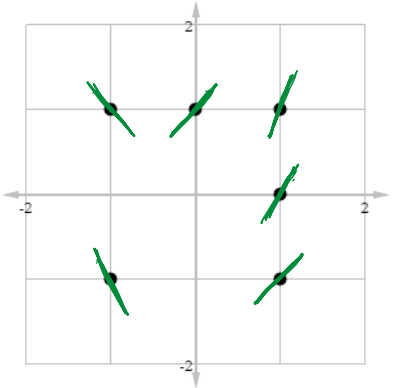
$-2 = C$

$y = \frac{1}{2}(\ln x)^2 - 2$



4. Sketch a slope field for the differential equation, $\frac{dy}{dx} = 2x + y$ for the indicated 6 points.

y \ x	-1	0	1
1	-1	1	3
0	$\times$	$\times$	2
-1	-3	$\times$	1



5. Match the differential equation to the appropriate slope field. (Don't forget to justify your answers)

(1) $\frac{dy}{dx} = x$ A

(2) $\frac{dy}{dx} = -\frac{x}{y}$ D

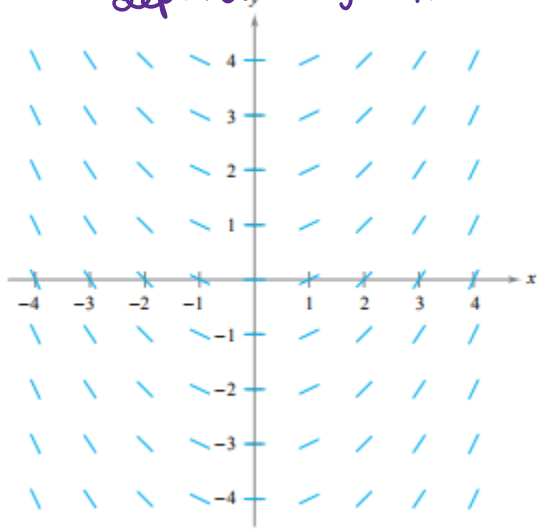
(3) $\frac{dy}{dx} = 4 - y$ C

(4) $\frac{dy}{dx} = 0.25x(4 - y)$ B

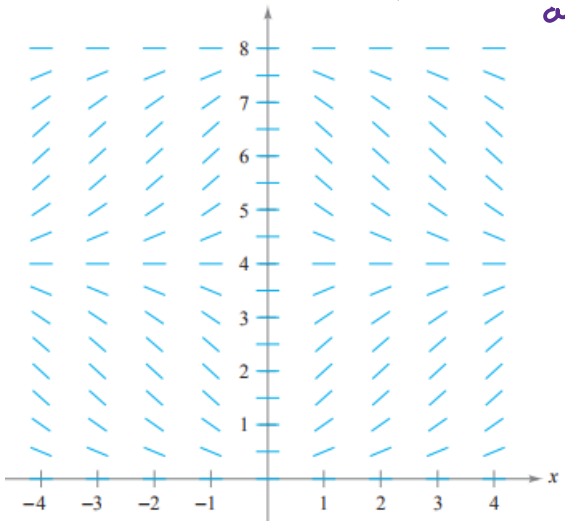
(sample reasons)

A.

depends only on x

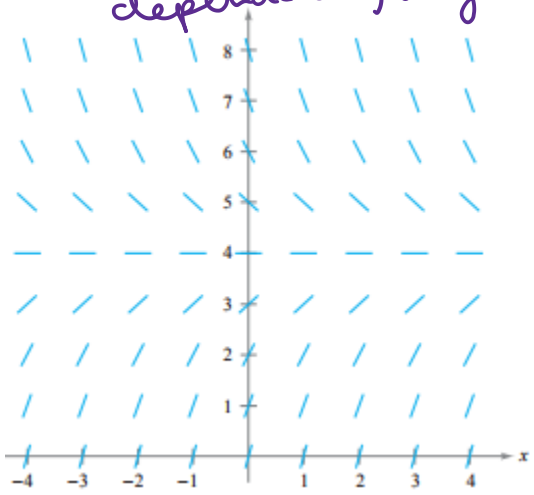


B. $\frac{dy}{dx} = 0$ when $y = 0$, $\frac{dy}{dx} > 0$ when $x > 0$ and $y < 4$

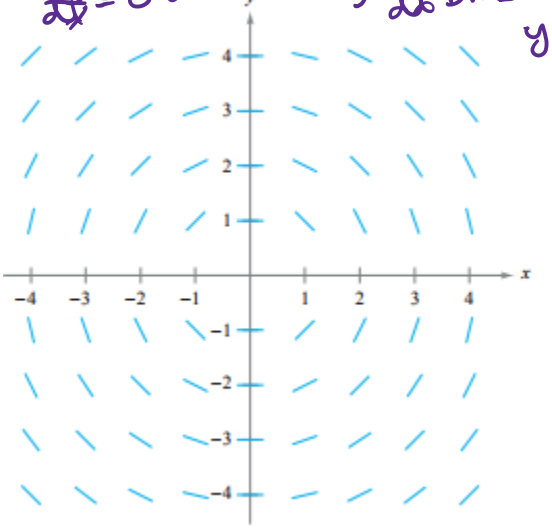


C.

depends only on y



D. $\frac{dy}{dx} = 0$ when $x = 0$, $\frac{dy}{dx}$ DNE when $y = 0$



6. Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$. Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(2) \approx 0$. Find the value of k.

$\Delta x = 1$

(x, y)	$\frac{dy}{dx}$	$\frac{dy}{dx}\Delta x$	$\frac{dy}{dx}\Delta x + y_1$
$(0, k)$	$2k+1$	$2k+1$	$2k+1+k = 3k+1$
$(1, 3k+1)$	$6k+6$	$6k+6$	$6k+6+3k+1 = 9k+7$
$(2, 9k+7)$			

$9k + 7 = 0$

$9k = -7$

$k = -\frac{7}{9}$