Some Practice Problems for Some Concepts in Unit 6

1. Using Euler's Method, approximate the particular solution to $y(0.3)$ of the differential equation $\frac{dy}{dx} = x + e^{-y}$ passing through $(0,1)$ using 3 increments of equal size. $\Delta x = \frac{0.3-0}{3} = 0.1$

(x, y)	$\frac{dy}{dx}$	$\frac{dy}{dx}(\Delta x)$	$\frac{dy}{dx}(\Delta x) + y_1$
$(0, 1)$	e^{-1}	$0.1e^{-1}$	$0.1e^{-1} + 1 = 1.037$
$(0.1, 1.037)$	0.455	0.045	$0.045 + 1.037 = 1.082$
$(0.2, 1.082)$	0.539	0.054	$0.05 + 1.082 = 1.136$
$(0.3, 1.136)$			

$\boxed{y(0.3) \approx 1.136}$

2. Solve the differential equation $\frac{dy}{dx} = \frac{x}{y}\sin x$ for $y(0) = 4$.

$\frac{dy}{dx} = \frac{x\sin x}{y}$

	u	dv
$+$	x	$\sin x$
$-$	1	$-\cos x$
	0	$-\sin x$

$\int y\,dy = \int x\sin x\,dx$

$\frac{1}{2}y^2 = -x\cos x + \sin x + C$

$\frac{1}{2}(4)^2 = -0\cos 0 + \sin 0 + C$

$8 = C$

$\frac{1}{2}y^2 = -x\cos x + \sin x + 8$

$y^2 = -x\cos x + \sin x + 16$

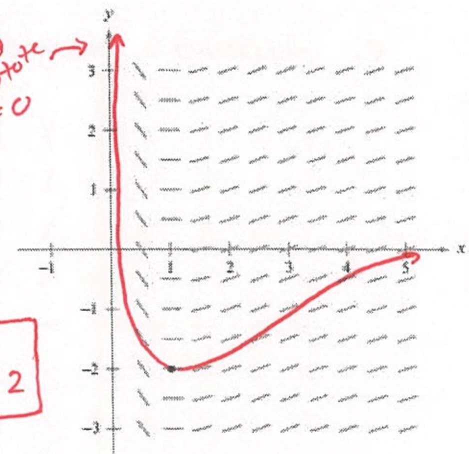
$y = \pm\sqrt{-x\cos x + \sin x + 16}$

$\boxed{y = \sqrt{-x\cos x + \sin x + 16}}$

☺ .. positive b/c $y = 4$, $y > 0$

3. Find and sketch the particular solution to $\frac{dy}{dx} = \frac{\ln x}{x}$ through the point $(1, -2)$.

vertical asymptote @ $x = 0$



$\frac{dy}{dx} = \frac{\ln x}{x}$

$\int dy = \int \frac{1}{x}\ln x\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$

$y = \int u\,du$

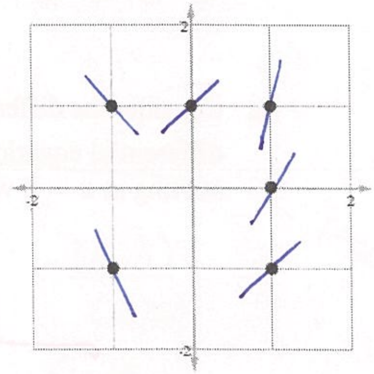
$y = \frac{1}{2}u^2 + C$

$y = \frac{1}{2}(\ln x)^2 + C \rightarrow \boxed{y = \frac{1}{2}(\ln x)^2 - 2}$

$-2 = \frac{1}{2}(\ln 1)^2 + C \rightarrow C = -2$

4. Sketch a slope field for the differential equation, $\frac{dy}{dx} = 2x + y$ for the indicated 6 points.

y \ x	-1	0	1
1	-1	1	3
0	X	X	2
-1	-3	X	1



5. Match the differential equation to the appropriate slope field. (Don't forget to justify your answers)

(1) $\frac{dy}{dx} = x$ A

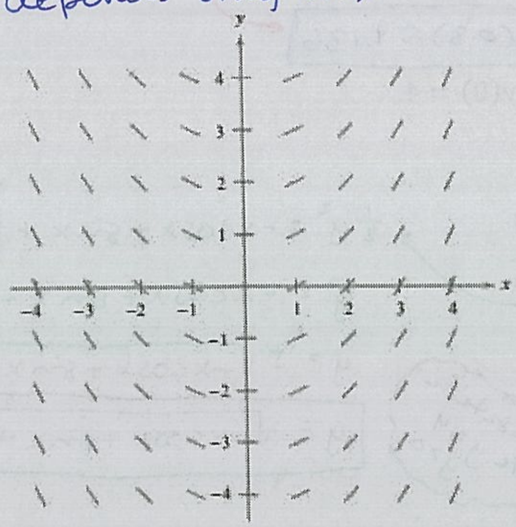
(2) $\frac{dy}{dx} = -\frac{x}{y}$ D

(sample reasons)

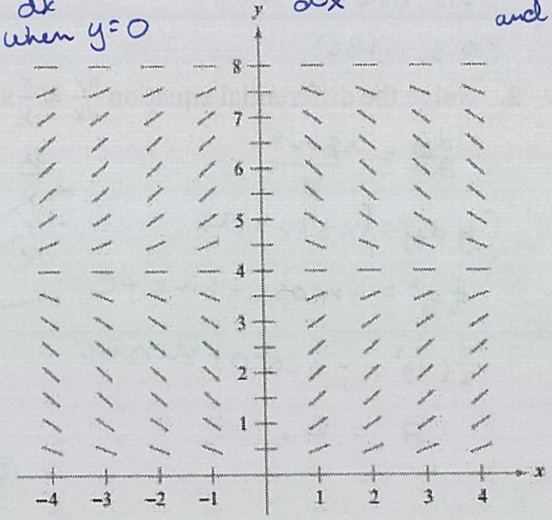
(3) $\frac{dy}{dx} = 4 - y$ C

(4) $\frac{dy}{dx} = 0.25x(4 - y)$ B

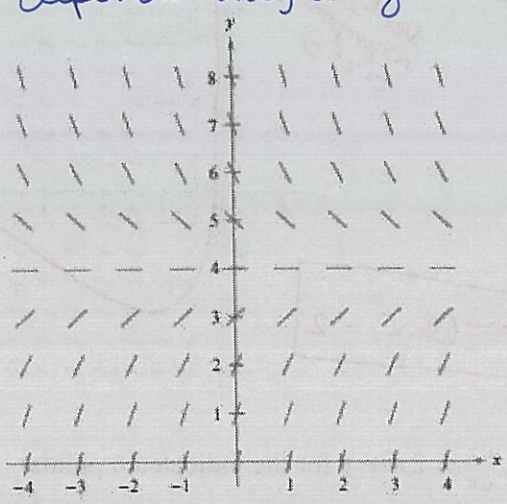
A. depends only on x



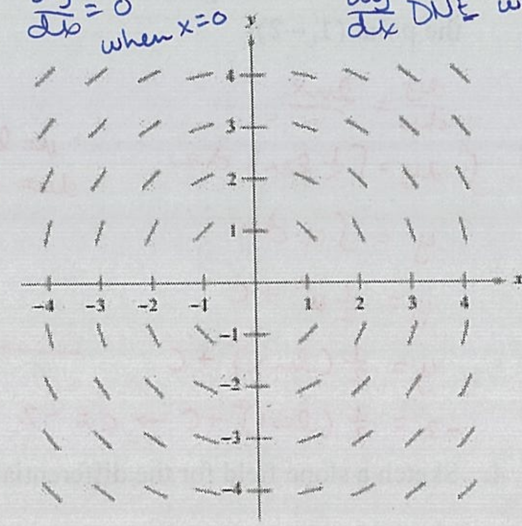
B. $\frac{dy}{dx} = 0$ when $y = 0$ $\quad$ $\frac{dy}{dx} > 0$ when $x > 0$ and $y < 4$



C. depends only on y



D. $\frac{dy}{dx} = 0$ when $x = 0$ $\quad$ $\frac{dy}{dx}$ DNE when $y = 0$



6. Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$. Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $\boxed{g(2) \approx 0}$. Find the value of k.

$\Delta x = 1$

(x, y)	dy/dx	$\frac{dy}{dx}(\Delta x)$	$\frac{dy}{dx}(\Delta x) + y_1$
$(0, k)$	$2k+1$	$2k+1$	$2k+1+k = 3k+1$
$(1, 3k+1)$	$6k+6$	$6k+6$	$6k+6+3k+1 = 9k+7$
$(2, 9k+7)$			

$g(2) \approx 0 = 9k+7 \rightarrow 9k+7=0$

$\boxed{k = -7/9}$