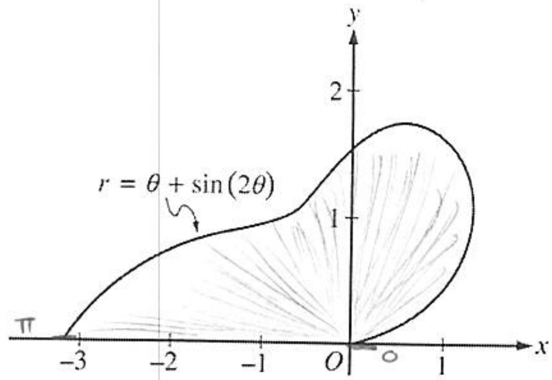
2005 AP® CALCULUS BC FREE-RESPONSE QUESTIONS



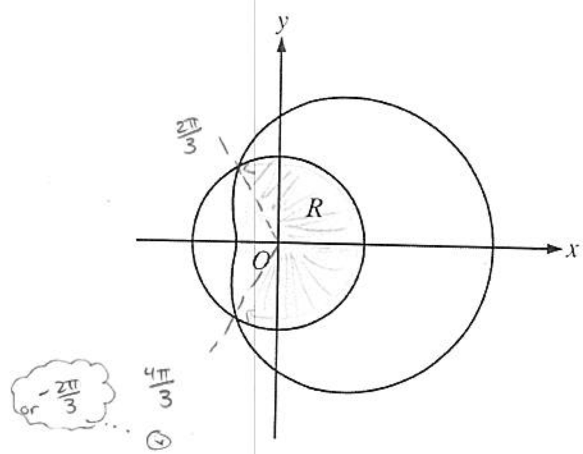
2. The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

   (a) Find the area bounded by the curve and the x-axis. from $\theta = 0$ to $\theta = \pi$

   $$\text{Area} = \frac{1}{2}\int_0^{\pi} (\theta + \sin 2\theta)^2 \, d\theta$$

   $$= \boxed{4.382}$$

2007 AP® CALCULUS BC FREE-RESPONSE QUESTIONS



3. The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

   (a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

Area of R = $\underbrace{\text{Area of Circle from } \theta = -2\pi/3 \text{ to } 2\pi/3}$ + Area of $r = 3 + 2\cos\theta$ from $\theta = \frac{2\pi}{3}$ to $\theta = \frac{4\pi}{3}$

$$= 2/3 \text{ of the entire circle} + \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (3 + 2\cos\theta)^2 \, d\theta$$

$$= \frac{2}{3}\pi(2)^2 + \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (3 + 2\cos\theta)^2 \, d\theta = \boxed{10.370}$$

2. An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

(b) Find the acceleration vector and the speed of the object at time $t = 1$.

(c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.

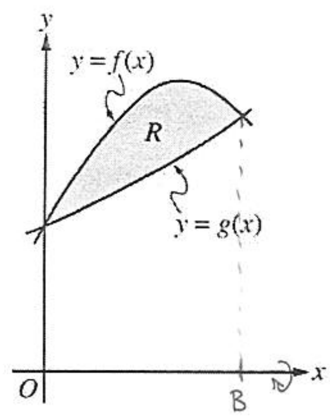
b) $a(t) = \langle x''(t), y''(t) \rangle$

$a(1) = \langle \frac{d}{dt}(\tan e^{-t})\Big|_{t=1}, \frac{d}{dt}(\sec e^{-t})\Big|_{t=1} \rangle = \boxed{\langle -.423, -.152 \rangle}$

Speed $= |v(t)| = \sqrt{(x'(t))^2 + (y'(t))^2}$

$= |v(1)| = \sqrt{(\tan e^{-1})^2 + (\sec e^{-1})^2} = \boxed{1.139}$

c) Distance $= \int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = \boxed{1.059}$



1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

$B = 1.136$

a) Area of $R = \int_0^B (f(x) - g(x))\, dx$

$= \int_0^B (1 + \sin 2x - e^{x/2})\, dx = \boxed{0.429}$

b) Volume $= \pi \int_0^B ((1+\sin 2x)^2 - (e^{x/2})^2)\, dx$

$= \boxed{4.267}$

c) Volume $= \int_0^B \frac{1}{2}\pi \left(\frac{f(x)-g(x)}{2}\right)^2 dx$

$= \boxed{0.078}$

diameter $= f(x) - g(x)$

radius $= \frac{f(x)-g(x)}{2}$

Area Semicircle $= \frac{1}{2}\pi r^2$