ICTM Math Contest Topics

Contest problems may include, but are not limited to, the following topics.

Algebra I

equations
inequalities
absolute value
simple matrices
slope
equations of lines
y-intercept
x-intercept
linear functions
intersection
union
quadratic functions
vertex
applications
geometry formulas
literal equations
functions
relations
function notation
integer exponents
simplifying radicals
radical equations
Pythagorean theorem
operations on polynomials
factoring
proportions
rational expressions
mean, mode, median
reading graphs

Geometry

logic
supplements
complements
congruency
probability
statistics
midpoints
properties of parallel lines
properties of perpendicular lines
properties of quadrilaterals
similarity
distance formula
midpoint formula
Pythagorean theorem
formulas involving circles
tangents
secants
chords
arc length
angle-arc theorems
inscribed polygons
circumscribed polygons
power theorems
area formulas
surface area
volume formulas
coordinate geometry
hero's formulas
inequalities
locus
incenter
orthocenter
centroid
Ptolemy's theorem
mass points
inradius
circumradius

Algebra 2

solving equations
inequalities
matrices
probability
graphs of lines
direct variation
inverse variation
fitting a line to data
frequency polygon
least squares
measures of central tendency
percentiles
absolute value
systems
conics
phase shift
reflections
symmetry
logarithms
relations
functions
inverse functions
combinatorics
roots of polynomials
graphing rational functions
asymptotes
matrices
rational exponents
radical equations
complex numbers
remainder theorem
factor theorem
synthetic division
rational-zeros theorem
relation of coefficients to roots
binomial theorem
operations of rational expressions
complex fractions
domain
range
exponential functions
sequences
series
applications

Precalculus

absolute value
rational exponents
factoring
roots of nth degree polynomials
coordinate geometry
complex numbers
functions
inverse functions
max & min problems
exponential functions
logarithmic functions
applications
trig functions
solving equations
- rational
- trig
- logarithmic
- complex #

law of cosines
law of sines
double angle formulas
addition formulas
vectors
DeMoivre's theorem
Gaussoan eliminations
matrices
determinants
systems
concis
rotation of axes
polar coordinates
remainder theorem
factor theorem
relation of coefficients to roots
conjugate roots
induction
binomial theorem
sequences
series
combinatorics
Descartes' rule of signs
normal curves
least squares line
probability
parametric equations
density functions
limits

ICTM Practice Roots & Zeroes of Polynomial Functions

Roots & Zeroes of Polynomial Functions

- Find roots & zeroes by dividing polynomials

  $x^3+3x^2+3x-63$

  Ex: Is 3 a root of the polynomial ~~x^3+3x^2 [illegible] $x-63$~~?

$$
\begin{array}{r}
x^2+6x+21 \\
x-3\,\overline{)\;x^3+3x^2+3x-63} \\
-(x^3-3x^2) \\
\hline
6x^2+3x \\
-(6x^2-18x) \\
\hline
21x-63 \\
-(21x-63) \\
\hline
0
\end{array}
$$

Since result is zero, then, 3 is a root of the polynomial.

- Synthetic Division

  $x^3+3x^2+3x-63$

  Ex: Is 3 a root of the polynomial ~~x^3+3x^2 [illegible] $x-63$~~?

$$
\begin{array}{r|rrrr}
3 & 1 & 3 & 3 & -63 \\
 & \downarrow & 3 & 18 & 63 \\
\hline
 & 1 & 6 & 21 & 0
\end{array}
$$

- When to use Calculator—AS MUCH AS POSSIBLE!!!!

- Remainder Theorem

  $x^3+3x^2+3x-63$

  Ex: Find the remainder when ~~x^3+3x^2 [illegible] $x-63$~~ is divided by $x - 3$.

$(+3)^3+3(+3)^2+3(+3)-63=0$

So, the remainder is 0.

Roots & Zeroes of Polynomial Functions

. If 3 is a root for x of $x^3 - 12x^2 + 47x + k = 0$, find the value of k.

2. Find the number of distinct times that the graph of $y = x^3 - x^2 - 14x + 24$ intersects the x-axis.

3. If 7 and -1 are two solutions for x in the equation $2x^3 + kx^2 - 44x + w = 0$, find the value of $k + w$.

4. When $y^2 + my + 8$ is divided by $y - 2$, the quotient is $f(y)$ and the remainder is k. When $y^2 + my + 8$ is divided by $y - 5$, the quotient is $g(y)$ and the remainder is p. If $k = p$, find the value of m.

5. Find the sum of the roots of $x^3 - 6x^2 + 11x - 6 = 0$

6. Given a cubic polynomial $p(x)$ whose reciprocal function $r(x) = 1/p(x)$ has asymptotes $x = 2$, $x = -3$, $x = 4$. If $p(0) = 6$, find $p(3)$.

7. Given that $\sqrt{3}$ is a root of $x^3 + 7x^2 - 3x - 21 = 0$, find the sum of the other two roots.

8. Given a polynomial with integral coefficients $x^3 + ax^2 + bx + c$. The sum of its zeroes is 1 and one zero is $\sqrt{2}$. Find the value of $a + b + c$.

Roots & Zeroes of Polynomial Functions: Solutions

1. –60
2. 3
3. –42
4. –7
5. 6
6. –3/2 or –1.5
7. $-7 - \sqrt{3}$
8. –1

ICTM Practice
Conic Sections

Conic Sections

- Parabolas

Vertical Axis of Symmetry

$(x-h)^2 = 4a(y-k)$
Directrix: $y = -a$
Focus: $(0, a)$
Vertex: (h, k)

Horizontal Axis of Symmetry

$(y-k)^2 = 4a(x-h)$
Directrix: $x = -a$
Focus: $(a, 0)$
Vertex: (h, k)

- Circles

$(x-h)^2 + (y-k)^2 = r^2$
Center: (h, k)
Radius: r

- Ellipses

Horizontal Major Axis

$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1, \quad a > b, \quad c^2 = a^2 - b^2$
Vertex: (h, k)
Length of major axis: $2a$
Length of minor axis: $2b$
Foci: $(h \pm c, k)$

Vertical Major Axis

$\dfrac{(y-k)^2}{a^2} + \dfrac{(x-h)^2}{b^2} = 1, \quad a > b, \quad c^2 = a^2 - b^2$
Vertex: (h, k)
Length of major axis: $2a$
Length of minor axis: $2b$
Foci: $(h, k \pm c)$

- Hyperbolas

Horizontal Axis of Symmetry

$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1, \quad c^2 = a^2 + b^2$
Vertices: $(h \pm a, k)$
Asymptotes: $y = \pm \dfrac{b}{a} x$

Vertical Axis of Symmetry

$\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1, \quad c^2 = a^2 + b^2$
Vertices: $(h, k \pm a)$
Asymptotes: $y = \pm \dfrac{a}{b} x$

Conic Sections

1. In interval notation, [k,w] is the domain for *x* of the real-valued conic section:
   $\frac{(x-3)^2}{16}+\frac{y^2}{25}=1$. Find the value of k + w.

2. Given the hyperbola $9x^2 - 16y^2 = 324$. Find the absolute value of the distance from the point (10,6) to the nearer asymptote of this hyperbola. Express your answer as an exact decimal.

3. The equation $\frac{x}{16}+\frac{y}{6}=1$ represents a(n):
   A. hyperbola
   B. parabola
   C. ellipse
   D. straight line
   E. rectangle

4. Find the length of the major axis of the ellipse $\frac{x^2}{49}+\frac{y^2}{25}=1$.

5. The two points (2,4) and (1,-3) lie on a circle, and the center of this circle lies on the line x + 2y = 0. The equation of this circle can be expressed in the form $(x-k)^2 + (y-w)^2 = p$. Find the value of k + w + p.

6. The equation of the circle in the first and fourth quadrants which has radius 1 and is tangent to both asymptotes of

$$\frac{x^2}{2}-\frac{y^2}{1}=1$$

   can be written in the form $(x-h)^2 + (y-k)^2 = 1$. Find the value of h + k. Express your answer in simplest radical form.

Conic Sections

~~Matrices & Determinants~~:Solutions

1. 6
2. 1.2
3. D
4. 14
5. 24
6. $\sqrt{3}$

ICTM Practice
Sequences & Series
Arithmetic & Geometric

Sequences & Series: Arithmetic & Geometric

Arithmetic Sequence

$a_n = a_{n-1} + d$ where d is common difference

or

$a_n = a_1 + d(n-1)$

Sum of an Arithmetic Sequence (Finite only)

$S_n = \frac{n}{2}(2a_1 + d(n-1))$ where d is common difference, a_1 is 1st term

Summation notation of an Arithmetic Series

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

Geometric Sequence

$a_n = a_{n-1}r$ where r is common ratio

Sum of a Geometric Sequence (Finite)

$S_n = \frac{a_1(1-r^n)}{1-r}$ where r is common ratio, a_1 is 1st term

Summation notation of an Geometric Series (Finite)

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

Sum of a Geometric Sequence(Infinite)

$$S_\infty = \frac{a_1}{1-r}$$

Summation notation of an Geometric Series(Infinite)

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + \ldots + a_k + \ldots \quad or \quad \sum_{k=1}^{\infty} a_k r^{k-1}$$

Sequences & Series: Arithmetic & Geometric

1. The arithmetic mean of a set of 50 numbers is 38. Two member of the set, namely 45 & 55, are discarded. Find the arithmetic mean of the remaining set of numbers.

2. Find the sum of the infinite series:

$$\frac{2}{3}+\frac{1}{9}+\frac{2}{27}+\frac{1}{81}+\frac{2}{243}+\frac{1}{729}+\ldots$$

3. The eighth term of an arithmetic sequence is five times the fourth term. The first term is 1. What is the second term? Give an exact answer.

4. If a recursive formula for a sequence is $t_n = t_{(n-1)} + 5$ and $t_1 = 2$. Find a formula that expresses t_n in terms of n. Write your answer in the form $t_n = a_m n^m + a_{(m-1)} n^{(m-1)} + \ldots + a_1 n + a^0$ where each a_k is a real number.

5. If the sum of the terms of a finite arithmetic sequence which begins –9, –6, –3,... is 66. Find the number of terms in the sequence.

6. The second term in a geometric series is 3/2. The sum of the first three terms is 21/4. Given that the fourth term is les than the third term, find the fourth term of this series.

7. Solve for x:

$$\sum_{n=1}^{x}(2n-6)=6$$

8. Find the exact value of

$$\sum_{n=1}^{\infty}\left(\frac{3^n}{4^{n-1}}\right)$$

9. The second term of an infinite geometric series is 4/3 and the sum of the geometric series is 6. Find the sum of the two distinct possible values for the first term of this series.

10. Solve for x:

$$\sum_{n=1}^{x}(4n+5)=372$$

Sequences & Series: Arithmetic & Geometric

Answers:

1. 75/2 or 37½ or 37.5
2. 7/8 or 0.875
3. ½ or 0.5
4. 5n – 3
5. 11
6. 3/8
7. 6
8. 12
9. 6
10. 12

ICTM Practice Trigonometry

Trigonometry

- SOH CAH TOA

- Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$ (where C is the angle included btn sides a & b)

- Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$\csc^2 x = 1 + \cot^2 x$

Double Angle Formulas

$$\sin 2u = 2\sin u\cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

or

$$\cos 2u = 1 - 2\sin^2 u$$

or

$$\cos 2u = 2\cos^2 u - 1$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Sum & Difference Formulas

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \pm \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

- Unit Circle
  Quadrant I

Angle	*Cos*	*Sin*	*Tan*
0	1	0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	0	1	0

Trigonometry

If k is a positive integer such that 27 < k < 35, find the sum of all distinct k such that sin(k)° is not a rational number.

2. For all values of θ and $\frac{\theta}{2}$ for which all trig values are defined,

$$\frac{1-\cos\theta-\tan^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}=\frac{k\cos\theta}{w+p\cos\theta}.$$ Find the value of *k* + *w* + *p*.

3. In scalene, but non-right Triangle ABC, tan(A) = $\frac{5}{2}$. If the length of each of the sides of Triangle ABC is an integer, find the smallest possible perimeter.

4. If $\sin(\theta)=\frac{12}{13}$ and $\cos(\theta)$ is negative, find $\tan(\theta)$. Express your answer as an exact decimal.

5. Find the largest possible perimeter of a triangle with two sides of respective lengths 18.64 and 20.88 and with an angle of 42°. Express your answer as a decimal rounded to 4 significant digits.

. For all real values of A, $\csc(A)\sec(A)=k(\csc(kA))$ where k>0. Find the value of k.

7. The range for y of $y=\cos\left(\frac{x}{7}\right)$ can be expressed as {y:k≤ y ≤ w}. Find the value of k – w.

8. If (-2,3) is on the terminal ray of an angle in standard position, then the sine of this angle can be expressed, in simplest radical form, as $\frac{k\sqrt{w}}{w}$. Find the value of k + w.

9. A lighthouse supervisor, 212 feet above the level surface of the water, spots a boat in the water at an angle of depression of 8°17'. Rounded to the nearest foot, find the number of feet that the boat is from the point, at water level, directly below the supervisor.

10. Find the value of $\cos^2(74.618°) + \sin^2(74.618°)$

Trigonometry: Solutions

1. 187
2. 4
3. 80
4. –2.4 (must be this decimal)
5. 70.12 (must be this decimal)
6. 2
7. –2
8. 16
9. 1456
10. 1

ICTM Practice Logarithms & Exponential Equations

Rules for Logarithms

- $\log_a x = b \Leftrightarrow a^b = x$
- Multiplication changes to addition: $\log_a(xy) = \log_a x + \log_a y$
- Division changes to subtraction: $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
- Exponents swing in front or coefficients become exponents: $\log_a x^y = y \log_a x$
- Change of base: $\log_a x = \frac{\log_b x}{\log_b a}$
- $\ln x = \log_e x$

Use rules to solve logarithmic or exponential equations.

ex: Simplify

$(\log_8 625)(\log_{25} 20)(\log_{20} 8)(\log_5 25)$

Solution: $\frac{\log 625}{\log 8} \frac{\log 20}{\log 25} \frac{\log 8}{\log 20} \frac{\log 25}{\log 5}$ change of base

$= \frac{\log 625}{\log 5}$ reduce

$= \frac{\log 5^4}{\log 5}$ rewrite as a power

$= \frac{4\log 5}{\log 5}$ swing exponent in front

$= 4$ reduce

ex: Solve for x: $2^{x^2+4x} = \frac{1}{8}$

Solution: $2^{x^2+4x} = \frac{1}{2^3}$ rewrite to get same bases

$2^{x^2+4x} = 2^{-3}$ rewrite

$x^2 + 4x = -3$ bases drop off, set exponents equal

$x^2 + 4x + 3 = 0$ Solve for x!

$(x+3)(x+1) = 0$

$x = -3, x = -1$

(check both answers to be certain)

Logarithms & Exponential Equations

1. Solve for n: $\dfrac{9^{1/2}(9^{2/3})}{9^{5/12}} = 3^n$

2. Find all values of x such that $(\log_3 x)^2 - \log_3(x^2) = 8$

3. Find all solutions of: $\log_2(\sin x) = \frac{1}{4}, \quad 0 \le x \le 2\pi$

4. Evaluate the product $(\log_9 11)(\log_{11} 13)(\log_{13} 15)\ldots(\log_{2183} 2185)(\log_{2185} 2187)$.

5. Find the largest solution of $\ln^2 x - \ln x^2 = 35$. Give an exact answer.

6. Find the largest value of x such that $(0.2)^{\cos 2x} \le 1$ and $0 \le x \le 90°$. Give your answer in degrees.

7. Solve:

$$\log_{100}(x + 2) = \log(x)$$

8. Solve for x:

$$\log_2(\log_x 2) = 2$$

9. Let k and w be two consecutive integers such that $k < x < w$. If $\log_7 143 = x$, find the value of $k+w$.

10. Find the value of x for which $\log_5 3 - \log_5 x + \log_5 7 = 1$. Express your answer as an improper fraction reduced to lowest terms.

11. If $243^{22} - 7 = 236 + \sum_{k=0}^{20} (\log_9((81x)^{(243^k)}))$ and if $x = 9^y$ where y is an integer, find the value of y.

Logarithms & Exponential Equations:Solutions

1. 3/2 or 1.5
2. 81, 1/9
3. No solutions
4. 7/2 or 3.5
5. e^7
6. 45°
7. 2
8. $2^{\frac{1}{4}}$ or $\sqrt[4]{2}$ or $2^{.25}$
9. 5
10. $\frac{21}{5}$
11. 58804

ICTM Practice Matrices & Determinants

Matrices & Determinants

- Operations on Matrices: "+","-","×"

Let $A = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 1 \\ 0 & -1 \end{bmatrix}$

ex: Find A + B   *ex:* Find A – B   *ex:* Find A×B   *ex:* Find B×A

- Determinants

2 × 2

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

3×3

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix}$$

Matrices & Determinants

1. If x is a positive integer and y is a negative integer, find the ordered pair for which x has the largest ossible value less than 100 such that the ordered pair satisfies the determinant equation:

$$\begin{vmatrix} 7 & x & 3 \\ 8 & y & 9 \\ 2 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 7 \\ 8 & x & 10 \\ 2 & y & 5 \end{vmatrix}$$. Be certain to express your answer as an ordered pair.

2. Find the product of the matrices: $\begin{bmatrix} 4 & 3 \\ 7 & 2 \end{bmatrix} \times \begin{bmatrix} 8 & 5 \\ 9 & 6 \end{bmatrix}$. Express your answer as a matrix.

3. Find as an improper fraction reduced to lowest terms, the absolute value of the difference of all distinct values of x for which: $\begin{vmatrix} x & 0 \\ 0 & \sqrt{3} \end{vmatrix}^2 + 2\begin{vmatrix} x & 2 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} 4 & -2 \\ 2 & 1 \end{vmatrix} = 0$

4. Solve for θ if θ is a radian measure such that $\frac{\pi}{4} < \theta < \pi$:

$$\begin{vmatrix} \sin(\theta) & -\sqrt{3} \\ -1/4 & \cos(\theta) \end{vmatrix} = 0$$

5. Solve: $\begin{vmatrix} x & -2 \\ 3 & x \end{vmatrix} = 30$

6. Solve $\begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \frac{\sqrt{3}}{2}$, $0 < \theta < \pi$, for the exact value of θ.

7. Given A = $\begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$, find A × B.

8. Given $A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $A_{(n+1)} = A_n \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$ for $n \geq 0$. Find "k" so that $A_k = \begin{bmatrix} 64 & 0 \\ 0 & 128 \end{bmatrix}$

Matrices & Determinants:Solutions

1. (89,-68)
2. $\begin{bmatrix} 59 & 38 \\ 74 & 47 \end{bmatrix}$
3. 10/3
4. $\pi/3$
5. $\pm 2\sqrt{6}$
6. $\pi/12$, $11\pi/12$ (must have both answers)
7. $\begin{bmatrix} 9 & -7 \\ 26 & -19 \end{bmatrix}$
8. 13