ROOTS + ZEROES OF POLYNOMIALS

① remainder theorem

$(3)^3 - 12(3)^2 + 47(3) + k = 0$

$60 + k = 0$

$k = \boxed{-60}$

② $y = x^3 - x^2 - 14x + 24$

graph it!

$\boxed{3}$ times

③ remainder theorem

$2(7)^3 + k(7)^2 - 44(7) + w = 0$

$378 + 49k + w = 0$

not needed

:)

$2(-1)^3 + k(-1)^2 - 44(-1) + w = 0$

$42 + k + w = 0$

$k + w = \boxed{-42}$

④ remainder theorem

$y^2 + my + 8$

$2^2 + m(2) + 8 = k$

$12 + 2m = k$

$y^2 + my + 8$

$5^2 + m(5) + 8 = p$

$33 + 5m = p$

$k = p$

$12 + 2m = 33 + 5m$

$-21 = 3m$

$\boxed{-7} = m$

⑤ $x^3 - 6x^2 + 11x - 6 = 0$

graph it!

$x = 1,\ x = 2,\ x = 3$

$1 + 2 + 3 = \boxed{6}$

⑥ $p(x) = k(x-a)(x-b)(x-c)$

$$r(x) = \frac{1}{k(x-2)(x+3)(x-4)}$$

$$r(0) = \frac{1}{k(-2)(3)(-4)} = \frac{1}{6}$$

$6 = 24k$

$\frac{1}{4} = k$

$p(3) = \frac{1}{4}(3-2)(3+3)(3-4)$

$= \boxed{-1.5}$ or $-3/2$

⑦ $\sqrt{3}, -\sqrt{3}$ are both roots

$x^3 + 7x^2 - 3x - 21 = 0$

graph it!

$x = -7$

$\boxed{-\sqrt{3} + -7}$ or $-7 - \sqrt{3}$

⑧ one zero → $\sqrt{2}$

another zero → $-\sqrt{2}$

$\sqrt{2} + -\sqrt{2} + r = 1$

$r = 1$ ← 3rd zero

remainder theorem

$x^3 + ax^2 + bx + c$

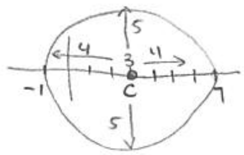
$1^3 + a(1)^2 + b(1) + c = 0$

$1 + a + b + c = 0$

$a + b + c = \boxed{-1}$

CONIC SECTIONS

① $\dfrac{(x-3)^2}{4^2} + \dfrac{y^2}{5^2} = 1$



$[-1, 7]$ Domain

$\uparrow$ k $\quad \uparrow$ w

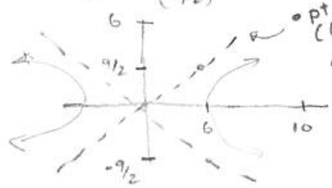
$k + w = -1 + 7$

$= \boxed{6}$

② $9x^2 - 16y^2 = 324$

$$\frac{x^2}{36} - \frac{y^2}{81/4} = 1$$

$$\frac{x^2}{6^2} - \frac{y^2}{(9/2)^2} = 1$$



pt $(10,6)$ closest to $y = \dfrac{9/2}{6}x$

$y = \frac{3}{4}x \rightarrow 4y = 3x$

$3x - 4y = 0$ ← standard form of eq. line

distance btn pt + line $= \dfrac{|ax + by + c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|3x - 4y + 0|}{\sqrt{3^2 + (-4)^2}}$ pt $(10,6)$

$= \dfrac{|3(10) - 4(6) + 0|}{5}$

$= \dfrac{6}{5}$

$= \boxed{1.2}$

③ $\dfrac{x}{16} + \dfrac{y}{6} = 1$

$6x + 16y = 96$

eq. of line

$\boxed{D}$

④ $\dfrac{x^2}{49} + \dfrac{y^2}{25} = 1$

$a^2 = 49$

$a = 7$

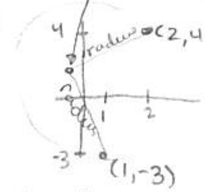
major axis $= 2a$

$= 2(7)$

$= \boxed{14}$

⑤ center (x, y) $\quad x + 2y = 0$, $y = -\frac{x}{2}$

$(x, -\frac{x}{2})$



to get radii, use distance formula $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\sqrt{(x-2)^2 + (-\frac{x}{2} - 4)^2} = \sqrt{(x-1)^2 + (-\frac{x}{2} - -3)^2}$

graph... $x = -2$

center $(-2, 1)$

radius $= \sqrt{(-2-2)^2 + (1-4)^2}$

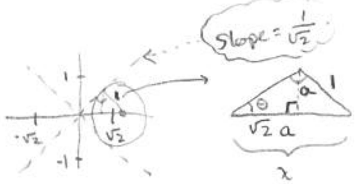
$= 5$

$(x - -2)^2 + (y - 1)^2 = 5^2$

$\uparrow$ $k = -2$ $\quad \uparrow$ $w = 1$ $\quad \uparrow$ $p = 25$

$k + w + p = -2 + 1 + 25 = \boxed{24}$

⑥



$\tan\theta = \dfrac{a}{\sqrt{2}a}$

$\tan\theta = \dfrac{1}{\sqrt{2}}$

$\theta = \tan^{-1}(\frac{1}{\sqrt{2}})$

$\sin\theta = \dfrac{1}{x}$

$\sin(\tan^{-1}(\frac{1}{\sqrt{2}})) = \dfrac{1}{x}$

$.5773 = \dfrac{1}{x}$

$x = 1.732$

$x = \boxed{\sqrt{3}}$

$\sqrt{3} = 1.732$
$\sqrt{2} = 1.414$

☺

SEQUENCES & SERIES: Arithmetic & Geometric

① $\frac{a_1+a_2+\cdots+a_{50}}{50} = 38$

$a_1+a_2+\cdots+a_{50} = 1900$

$\frac{a_1+a_2+\cdots+a_{50}-45-55}{48}$

$\frac{1900-45-55}{48} = \boxed{37.5}$

② $\frac{2}{3}+\frac{1}{9}+\frac{2}{27}+\frac{1}{81}+\frac{2}{243}+\frac{1}{729}+\cdots$

$\frac{2}{3}+\frac{2}{27}+\frac{2}{243}+\cdots + \frac{1}{9}+\frac{1}{81}+\frac{1}{729}+\cdots$ (regroup)

$\frac{2}{3}+\frac{2}{3^3}+\frac{2}{3^5}$ $\qquad$ $\frac{1}{9}+\frac{1}{9^2}+\frac{1}{9^3}$

$S_\infty = \frac{2/3}{1-1/9}$ $\qquad$ $S_\infty = \frac{1/9}{1-1/9}$

$S_\infty = 3/4$ $\qquad$ $S_\infty = 1/8$

$3/4 + 1/8 = \boxed{7/8}$

③ $a_8 = 5a_4 \qquad a_1 = 1 \qquad a_2 = ?$

$a_8 = a_1 + (8-1)d \qquad a_4 = a_1 + (4-1)d$

$5a_4 = 1+7d \qquad a_4 = 1+3d$

$5(1+3d) = 1+7d$

$5+15d = 1+7d$

$4 = -8d$

$-\frac{1}{2} = d$

$a_2 = a_1 + (2-1)d$

$= 1 + 1(-\frac{1}{2})$

$a_2 = \boxed{\frac{1}{2}}$

④ $t_n = t_{n-1}+5, \quad t_1 = 2$

$t_2 = t_1 + 5 = 2+5 = 7$

$t_3 = t_2 + 5 = (2+5)+5 = 2+2(5)$ $\quad 3-1$

$t_4 = t_3 + 5 = (2+2(5))+5 = 2+3(5)$ $\quad 4-1$

$\vdots$

$t_n = 2+(n-1)5$

$= 2+5n-5$

$= \boxed{5n-3}$

⑤ $a_1 = -9$

$d = -6-(-9)$

$= 3$

$S_n = \frac{n}{2}(a_1+a_n)$ where $a_n = a_1+(n-1)d$

$66 = \frac{n}{2}(-9+(-9)+(n-1)3)$

$66 = \frac{n}{2}(-18+3n-3)$

$66 = \frac{n}{2}(-21+3n)$

$132 = -21n+3n^2$

$0 = 3n^2-21n-132$

$0 = 3(n^2-7n-44)$

$0 = 3(n+4)(n-11)$

$n = \boxed{11}$

OR solve it any time w/ graph calc.

⑥ $a_2 = \frac{3}{2}$ $\qquad a_1+a_2+a_3 = \frac{21}{4} \qquad a_4 < a_3$

$a_2 = a_1 r$

$\frac{3}{2} = a_1 r$

$\frac{3}{2r} = a_1$

$S_3 = \frac{a_1(1-r^3)}{1-r} = \frac{21}{4}$

$\frac{\frac{3}{2r}(1-r^3)}{1-r} = \frac{21}{4}$

$\frac{3}{2r} - \frac{3r^2}{2} = \frac{21(1-r)}{4}$

graph it

$r = .5, \quad r = 2$

keep since $a_4 < a_3$ if r is less than 1, a_4 will be less than a_3

$a_2 = a_1 r$

$a_3 = a_2 r$

$a_4 = a_3 r$

$= a_2 r^2$

$= (\frac{3}{2})(\frac{1}{2})^2$

$= \boxed{3/8}$

(7) $\sum_{n=1}^{x}(2n-6)=6$

use calculator, with different values of x.

$x=\boxed{6}$

$\sum_{n=1}^{6}(2n-6)=6$

$-4+-2+0+2+4$

☺

(8) $\sum_{n=1}^{\infty}\left(\frac{3^n}{4^{n-1}}\right)=\frac{3}{4^0}+\frac{3^2}{4^1}+\frac{3^3}{4^2}+\ldots=3+\frac{9}{4}+\frac{27}{16}$

$r=3/4$

$S_\infty=\frac{a_1}{1-r}$

$=\frac{3}{1-3/4}$

$=\boxed{12}$

(9) $a_2=\frac{4}{3}$

$a_2=a_1r$

$\frac{4}{3}=a_1r$

$\frac{4}{3a_1}=r$

$S_\infty=6$

$\frac{a_1}{1-r}=6$

$\frac{a_1}{1-\frac{4}{3a_1}}=6$

graph it

$a_1=2$ $a_1=4$

$2+4=\boxed{6}$

(10) $\sum_{n=1}^{x}(4n+5)=372$

put into y=

(look at table of values

1	9
2	22
3	39
⋮	
10	270
11	319
12	372

$\boxed{12}$ → 12

TRIGONOMETRY

① Put sin x in [y=]

Tblset 27

ΔTbl = 1

Look at table

28	.46947
29	.48481
30	.5 ← rational # $\frac{1}{2}$
31	.515
32	.5299
33	.5446
34	.5591

Sum = 28 + 29 + 31 + 32 + 33 + 34

$= \boxed{187}$

② $\dfrac{1-\cos\theta - \tan^2\left(\frac{\theta}{2}\right)}{\sin^2\left(\frac{\theta}{2}\right)}$

$\dfrac{1-\cos\theta}{\sin^2(\theta/2)} - \dfrac{1}{\cos^2(\theta/2)}$

$\dfrac{1-\cos\theta}{\frac{1}{2}(1-\cos\theta)} - \dfrac{1}{\frac{1}{2}(1+\cos\theta)}$

$\dfrac{1}{1/2} - \dfrac{1}{1/2(1+\cos\theta)}$

$2 - \dfrac{2}{1+\cos\theta}$

$\dfrac{2(1+\cos\theta) - 2}{1+\cos\theta}$

$\dfrac{2+2\cos\theta-2}{1+\cos\theta} = \dfrac{2\cos\theta}{1+\cos\theta}$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(1-\cos 2\left(\frac{\theta}{2}\right)\right)$

$= \frac{1}{2}(1-\cos\theta)$

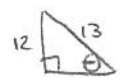
$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$\cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(1+\cos 2\cdot\frac{\theta}{2}\right)$

$= \frac{1}{2}(1+\cos\theta)$

$\left.\begin{array}{l} k=2 \\ w=1 \\ p=1 \end{array}\right\}$ 2 + 1 + 1 $\boxed{4}$

③ :(

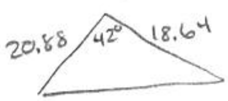
④ 

$\sin\theta = \frac{12}{13}$

-5 since $\cos\theta$ is negative

$\tan\theta = \frac{12}{-5}$

$= \boxed{-2.4}$

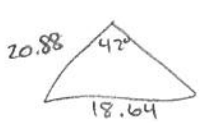
⑤ 

Law of cosines

$c^2 = (18.64)^2 + (20.88)^2 - 2(18.64)(20.88)\cos 42°$

$c = 14.316\ldots$

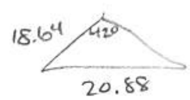
P = 53.84



$(18.64)^2 = (20.88)^2 + a^2 - 2(20.88)(a)\cos 42°$

a = 3.18 or 27.86

P = 42.70 or 67.38



$(20.88)^2 = 18.64^2 + a^2 - 2(18.64)a\cos 42°$

a = 30.597

P = $\boxed{70.12}$

Largest perimeter

⑥ $\csc A \sec A = k(\csc(kA))$

$\frac{1}{\sin A} \cdot \frac{1}{\cos A} = k \frac{1}{\sin kA}$

$\frac{1}{2}\left(\frac{1}{\sin A \cos A}\right) = \frac{1}{2}\left(\frac{k}{\sin kA}\right)$

$\frac{1}{2\sin A \cos A} = \frac{k}{2\sin kA}$

$\frac{1}{\sin 2A} = \frac{k}{2\sin kA}$ → $\frac{k}{2} = 1$

$k = 2$

$k = 2$

so, $\boxed{k = 2}$

⑦ graph $y = \cos\left(\frac{x}{7}\right)$

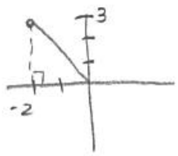
Range: $-1 \le y \le 1$

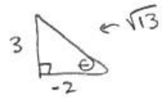
k ↑ ↑ w

$k - w = -1 - 1$

$= \boxed{-2}$

⑧



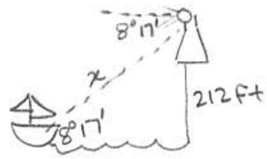


$\sin\theta = \frac{3}{\sqrt{13}}$

$= \frac{3\sqrt{13}}{13}$

$k = 3$
$w = 13$
$k + w = \boxed{16}$

⑨



$\tan 8°17' = \frac{212}{x}$

$x = 1456$ ft

$\boxed{1456}$

⑩ $\cos^2(74.618) + \sin^2(74.618)$

$\boxed{1}$ $\sin^2 x + \cos^2 x = 1$

☺

LOGARITHMS + EXPONENTIAL EQUATIONS

① $\dfrac{9^{1/2}\cdot 9^{2/3}}{9^{5/12}} = 3^n$

$9^{1/2+2/3-5/12} = 3^n$

$9^{3/4} = 3^n$

$(3^2)^{3/4} = 3^n$

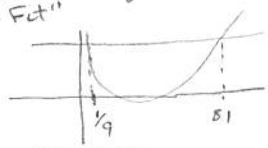
$3^{3/2} = 3^n$

$\boxed{3/2} = n$

② $(\log_3 x)^2 - \log_3(x^2) = 8$

graph it ... get intersections

try "Zoom Fit"

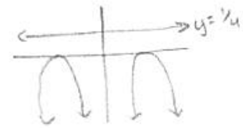


$x = \boxed{\frac{1}{9}, 81}$

③ $\log_2(\sin x) = \frac{1}{4}$

graph it! (radians)

no intersection



$\boxed{\text{no solution}}$

④ $(\log_9 11)(\log_{11} 13)(\log_{13} 15)\ldots(\log_{2183} 2185)(\log_{2185} 2187)$

$\left(\frac{\log 11}{\log 9}\right)\left(\frac{\log 13}{\log 11}\right)\left(\frac{\cancel{\log 15}}{\log 13}\right)\ldots\left(\frac{\log 2185}{\log 2183}\right)\left(\frac{\log 2187}{\cancel{\log 2185}}\right)$

$\frac{\log 2187}{\log 9}$

$\boxed{3.5}$ or $\frac{7}{2}$

⑤ $\ln^2 x - \ln x^2 = 35$

$\ln^2 x - 2\ln x = 35$

$\ln^2 x - 2\ln x - 35 = 0$

$(\ln x)^2 - 2\ln x - 35 = 0$ 　 $u = \ln x$ ☺

$u^2 - 2u - 35 = 0$

$(u-7)(u+5) = 0$

$u = 7, \ u = -5$

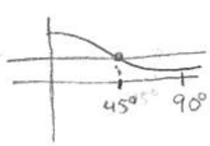
$\ln x = 7, \ \ln x = -5$

$x = e^7, \ x = e^{-5}$

$\boxed{e^7}$ is Largest

⑥ $(0.2)^{\cos x} \le 1$

graph it! (degrees)



(not to scale)

$\boxed{45°}$

⑦ $\log_{100}(x+2) = \log x$

$\frac{\log(x+2)}{\log 100} = \log x$

$\frac{\log(x+2)}{\log 10^2} = \log x$

$\frac{\log(x+2)}{2} = \log x$

$\log(x+2) = 2\log x$

$\log(x+2) = \log x^2$

$x + 2 = x^2$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = \boxed{2}$, ~~$x = -1$~~ doesn't check

⑧ $\log_2(\log_x 2) = 2$

$\cancel{2}^{\cancel{\log_2}(\log_x 2)} = 2^2$

$\log_x 2 = 4$

$\cancel{x}^{\cancel{\log_x} 2} = x^4$

$2 = x^4$

$\boxed{\sqrt[4]{2}} = x$

⑨ $k < \log_7 143 < w$

$k < 2.5503 < w$

$2 < 2.5503 < 3$

$k = 2,\ w = 3$

$k + w = 2 + 3$

$= \boxed{5}$

⑩ $\log_5 3 - \log_5 x + \log_5 7 = 1$

$\log_5\left(\frac{3}{x} \cdot 7\right) = 1$

$5^{\log_5\left(\frac{21}{x}\right)} = 5^1$

$\frac{21}{x} = 5$

$5x = 21$

$x = \boxed{\frac{21}{5}}$

⑪ $243^{22} - 7 = 236 + \sum_{k=0}^{20} \log_9\left((81x)^{243^k}\right)$ $\quad x = 9^y$

$243^{22} - 7 = 236 + \sum_{k=0}^{20} \log_9 (81 \cdot 9^y)^{243^k}$

$243^{22} - 7 = 236 + \sum_{k=0}^{20} 243^k \cdot \log_9 (81 \cdot 9^y)$

$243^{22} - 7 = 236 + \sum_{k=0}^{20} 243^k \cdot (\log_9 81 + \log_9 9^y)$

$243^{22} - 7 = 236 + (2+y) \sum_{k=0}^{20} 243^k$

$$\frac{243^{22} - 7 - 236}{\sum_{k=0}^{20} 243^k} = 2 + y$$

$58806 = 2 + y$

$\boxed{58804} = y$

MATRICES + DETERMINANTS

①
$$\begin{matrix} 7 & x & 3 & 7 & x \\ 8 & y & 9 & 8 & y \\ 2 & 1 & 5 & 2 & 1 \end{matrix} = \begin{matrix} 1 & 2 & 7 & 1 & 2 \\ 8 & x & 10 & 8 & x \\ 2 & y & 5 & 2 & y \end{matrix}$$

$35y + 18x + 24$

$- (4y + 63 + 40x)$

$5x + 40 + 56y$

$- (14x + 10y + 80)$

$$29y - 22x - 39 = -9x + 46y - 40$$

$$-17y = 13x - 1$$

$$y = \frac{13x-1}{-17}$$

graph + table start @ x=100 and go down

find 1st y-value that is an integer

$\boxed{(89, -68)}$

② calculator

$$\boxed{\begin{bmatrix} 59 & 38 \\ 74 & 47 \end{bmatrix}}$$

③ $\begin{vmatrix} x & 0 \\ 0 & \sqrt{3} \end{vmatrix}^2 + 2\begin{vmatrix} x & 2 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} 4 & -2 \\ 2 & 1 \end{vmatrix} = 0$

$(\sqrt{3}x)^2 + 2(x) - (4 - (-2)) = 0$

$3x^2 + 2x - 8 = 0$

graph... $x = -2$, $x = 1.333$ or $\frac{4}{3}$

$\left|-2 - \frac{4}{3}\right| = \boxed{\frac{10}{3}}$

④ $\begin{vmatrix} \sin\theta & -\sqrt{3} \\ -1/4 & \cos\theta \end{vmatrix} = 0$ $\quad \pi/4 < \theta < \pi$

$\sin\theta\cos\theta - \frac{\sqrt{3}}{4} = 0$

$\sin\theta\cos\theta = \frac{\sqrt{3}}{4}$

graph it in degrees... set window to $[\pi/4, \pi]$

$\theta = 60°$ convert to radians

$\theta = \boxed{\pi/3}$

⑤ $\begin{vmatrix} x & -2 \\ 3 & x \end{vmatrix} = 30$

$x^2 - (-6) = 30$

$x^2 = 24$

$x = \pm\sqrt{24}$

$x = \boxed{\pm 2\sqrt{6}}$

⑥ $\begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \frac{\sqrt{3}}{2}$ $\quad 0 < \theta < \pi$

$\cos^2\theta - \sin^2\theta = \frac{\sqrt{3}}{2}$

$\cos 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = \pi/6$ or $2\theta = \frac{11\pi}{6}$

$\theta = \boxed{\frac{\pi}{12} \text{ or } \frac{11\pi}{12}}$ both are in interval

⑦ $\begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix} \times \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$

calculator

$$\boxed{\begin{bmatrix} 9 & -7 \\ 26 & -19 \end{bmatrix}}$$

⑧ $A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ $\quad A_{(n+1)} = A_n\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$

$A_2 = A_1\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

$A_3 = A_2\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

$A_4 = A_3\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$

notice pattern, so $A_5 = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$ or $\begin{bmatrix} 2^2 & 0 \\ 0 & 2^3 \end{bmatrix}$ $2+3=5$

$A_6 = \begin{bmatrix} 0 & 8 \\ 8 & 0 \end{bmatrix}$

⋮

so $k = \boxed{13}$ ← $A_{13} = \begin{bmatrix} 64 & 0 \\ 0 & 128 \end{bmatrix}$ or $\begin{bmatrix} 2^6 & 0 \\ 0 & 2^7 \end{bmatrix}$ $6+7=13$