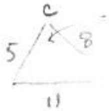
JR/SR 8 Person
2003

(1) — largest ∠



$c^2 = a^2 + b^2 - 2ab\cos C$

$11^2 = 8^2 + 5^2 - 2(5)(8)\cos C$

$121 = 64 + 25 - 80\cos C$

$121 = 89 - 80\cos C$

$32 = -80\cos C$

$\frac{32}{-80} = \cos C$

$\boxed{\frac{2}{-5} = \cos C}$

(2) Area of ellipse $= \pi ab$

$= \pi(8)(6)$

$= 48\pi$

Area of circle $= \pi r^2$

$= \pi(6)^2$

$= 36\pi$

$\frac{36\pi}{48\pi} = \boxed{\frac{3}{4}}$

(3) $x = -2 + 3i\sqrt{2}$ $\quad x = -2 - 3i\sqrt{2}$

$x + 2 - 3i\sqrt{2} = 0$ $\quad x + 2 + 3i\sqrt{2} = 0$

$(x + 2 - 3i\sqrt{2})(x + 2 + 3i\sqrt{2}) = 0$

$x^2 + 2x + 3i\sqrt{2}x + 2x + 4 + 6i\sqrt{2} - 3i\sqrt{2}x - 6i\sqrt{2} - 18i^2$

$x^2 + 4x + 4 + 18 = 0$

$x^2 + 4x + 22 = 0$

$$\begin{array}{r} 2x - 1 \\ x^2+4x+22 \enclose{longdiv}{2x^3 + 7x^2 + ax + b} \\ -(2x^3 + 8x^2 + 44x) \\ -x^2 + ax - 44x + b \\ -(-x^2 - 4x - 22) \\ ax - 40x + b + 22 \end{array}$$

$(a-40)x + b + 22 = 0$

$a - 40 = 0 \quad b + 22 = 0$

$a = 40 \quad b = -22$

$\boxed{(40, -22)}$

(4) $\frac{x^3+5}{x+2}$

if denominator $= 1$ or -1, integer: $\frac{(-1)^3+5}{-1+2}$ ✓ $\quad \frac{(-3)^3+5}{-3+2}$ ✓

if denominator $= 3$ or -3, integer: $\frac{1^3+5}{1+2} = \frac{6}{3}$ ✓ $\quad \frac{(-5)^3+5}{-5+2} = \frac{-120}{-3}$ ✓

$\boxed{x = -1, -3, 1, -5}$

(5)
$$\begin{array}{ccccc} 1 & 0 & -1 & 1 & 0 \\ y & 2 & 1 & y & 2 \\ y & 3 & x & y & 3 \end{array} = 5$$

$-(-2y + 3 + 0) + 2x + 0 - 3y$

$2y - 3 + 2x - 3y = 5$

$-y = -2x + 8$

$\boxed{y = 2x - 8}$

(6) $a_1 = \log_4 3 \quad d = \frac{1}{2}$

$a_{10} = \log_4 a$

$a_{10} = a_1 + (n-1)d$

$= \log_4 3 + (10-1)(\frac{1}{2})$

$= \log_4 3 + 9(\frac{1}{2})$

$= \log_4 3 + \frac{9}{2}$

$= \log_4 3 + \log_4 4^{9/2}$

$= \log_4 3 + \log_4 (\sqrt{4})^9$

$= \log_4 3 + \log_4 2^9$

$= \log_4 3 + \log_4 512$

$= \log_4 (3 \cdot 512)$

$= \log_4 (1536)$

$\boxed{a = 1536}$

(7) $a_1 = 2$

$a_2 = 2a_1 + 3$
$= 2(2) + 3 = 7$

$a_3 = 2a_2 + 3$
$= 2(7) + 3 = 17$

$a_4 = 2a_3 + 3$
$= 2(17) + 3 = 37$

$a_5 = 2a_4 + 3$
$= 2(37) + 3 = 77$

$\sum_{k=1}^{5} a_k = a_1 + a_2 + a_3 + a_4 + a_5$

$= 2 + 7 + 17 + 37 + 77$

$= 9 + 110 + 21$

$= \boxed{140}$

(8) ARRAN$\boxed{GE}$ → could be GE or EG

$\underbrace{\text{- - - - -}}_{\text{6 items}}$ — A → repeated, R → repeated

$2\left(\frac{6!}{2!2!}\right)$

$2\left(\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{2 \cdot 2}\right) = 2(6 \cdot 5 \cdot 2 \cdot 3)$

$= \boxed{360}$

(9) $7x + 2y = 31 \rightarrow 7x + 2y = 31$

$-x + 4y = -13 \rightarrow -7x + 28y = -91$

$30y = -60$

$y = -2$

$-x + 4(-2) = -13$

$-x - 8 = -13$

$-x = -5$

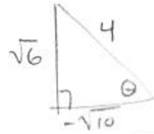
$x = 5$

$\boxed{(5, -2)}$

(10) $\sin\theta = \frac{\sqrt{6}}{4}$

$\frac{\pi}{2} \le \theta \le \pi$

Quad II
(cos negative)



$(\sqrt{6})^2 + a^2 = 4^2$

$6 + a^2 = 16$

$a^2 = 10$

$a = \sqrt{10}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}}$

$= \frac{4}{-\sqrt{10}}$

$= -\frac{4\sqrt{10}}{10} = \boxed{\frac{-2\sqrt{10}}{5}}$

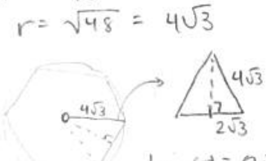
(11) $x^2 + 8x + \quad + y^2 - 6y \quad = 23$

$x^2 + 8x + 16 + y^2 - 6y + 9 = 23 + 16 + 9$

$(x+4)^2 + (y-3)^2 = 48$

$r^2 = 48$

$r = \sqrt{48} = 4\sqrt{3}$



height $= (2\sqrt{3})(\sqrt{3})$
$= 6$

$A = \left(\frac{1}{2}(6)(4\sqrt{3})\right)6 = \boxed{72\sqrt{3}}$

(12) $x = 4 - 6t$

$x - 4 = -6t$

$\frac{x-4}{-6} = t$

$y = 3 + 2t$

$y = 3 + 2\left(\frac{x-4}{-6}\right)$

$= 3 - \frac{1}{3}x + \frac{4}{3}$

$m = \boxed{-\frac{1}{3}}$

(13) $\sin 40° = \cos 50°$

$\sin 157° = \sin 23°$

$\sin 40° \cos 23° - \sin 50° \sin 157° =$

$\cos 50° \cos 23° - \sin 50° \sin 23°$

$= \cos(50 + 23)$

$= \cos(73°)$

$x = \boxed{73}$

(14) $\log_2 x + \log_3 y = 4 \rightarrow \log_2 x + \log_3 y = 4$

$\log_x 2 + \log_y 3 = 1 \rightarrow \dfrac{\log_2 2}{\log_2 x} + \dfrac{\log_3 3}{\log_3 y} = 1$

Let $\log_2 x = a$

$\log_3 y = c$

$a + c = 4 \rightarrow c = 4 - a$

$\frac{1}{a} + \frac{1}{c} = 1 \rightarrow \frac{1}{a} + \frac{1}{4-a} = 1$

$4 - a + a = a(4-a)$

$4 = 4a - a^2$

$a^2 - 4a + 4 = 0$

$(a-2)^2 = 0$

$a = 2$

$c = 4 - a$

$c = 4 - 2$

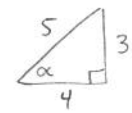
$c = 2$

$\log_2 x = 2 \qquad \log_3 y = 2$

$x = 4 \qquad y = 9$

$\boxed{(4,9)}$

(15) $\cos(\underbrace{2\sin^{-1}(\tfrac{3}{5})}_{\alpha})$



$\cos(2 \cdot \alpha)$

$= \cos^2 \alpha - \sin^2 \alpha$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \boxed{\frac{7}{25}}$

(16) $\sum_{k=1}^{\infty} \left(\frac{2x+3}{2}\right)^k$ geometric series

$|r| < 1$

$\left|\frac{2x+3}{2}\right| < 1$

$-1 < \frac{2x+3}{2} < 1$

$-2 < 2x + 3 < 2$

$-5 < 2x < -1$

$\boxed{-\frac{5}{2} < x < -\frac{1}{2}}$

(17) $y = -2x^2 + 7x - k$

$y = -2\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) - k + \frac{49}{8}$

$y = -2\left(x - \frac{7}{4}\right)^2 \underbrace{- k + \frac{49}{8}}$

$-k + \frac{49}{8} < 0$

$-k < -\frac{49}{8}$

$\boxed{k > \frac{49}{8}}$

(18) $\sin(4\theta) + \cos 2\theta = 0$

$\sin(2 \cdot 2\theta) + \cos 2\theta = 0$

$2\sin 2\theta \cos 2\theta + \cos 2\theta = 0$

$\cos 2\theta(2\sin 2\theta + 1) = 0$

$\cos 2\theta = 0 \qquad 2\sin 2\theta + 1 = 0$

$2\theta = 90°, 270° \qquad \sin 2\theta = -\frac{1}{2}$

$\theta = 45°, 135° \qquad 2\theta = \sin^{-1}(-\frac{1}{2})$

$2\theta = 210°, 330°$

$\theta = 105°, 165°$

$\boxed{45°, 105°, 135°, 165°}$

(19) $A^{-1} = \frac{1}{\det A}\begin{bmatrix} -3 & 4 \\ -x & 3 \end{bmatrix}$

$= \frac{1}{-9+4x}\begin{bmatrix} -3 & 4 \\ -x & 3 \end{bmatrix}$

$\begin{bmatrix} 3 & -4 \\ x & -3 \end{bmatrix} = \begin{bmatrix} \frac{-3}{-9+4x} & \frac{4}{-9+4x} \\ \frac{-x}{-9+4x} & \frac{3}{-9+4x} \end{bmatrix}$

$3 = \frac{-3}{-9+4x}$

$-27 + 12x = -3$

$12x = 24$

$x = 2$

$\boxed{2}$

(20) $f(x) = 3^{4-x^2}$

max when $4 - x^2$ is largest

so, if $x = 0$, $4 - (0)^2 = 4$

$f(0) = 3^{4-0^2}$

$= 3^4$

$= \boxed{81}$