8-PERSON
LEMONT 2005

① $\frac{\log 8}{\log 243}\left(\frac{\log 81}{\log 32} - \frac{\log 27}{\log 4}\right)$

$\frac{\log 2^3}{\log 3^5}\left(\frac{\log 3^4}{\log 2^5} - \frac{\log 3^3}{\log 2^2}\right)$

$\frac{3\log 2}{5\log 3}\left(\frac{4\log 3}{5\log 2} - \frac{3\log 3}{2\log 2}\right)$

$\frac{12}{25} - \frac{9}{10}$

$\frac{24}{50} - \frac{45}{50}$

$\boxed{-\frac{21}{50}}$

② $(x+2)^{x^2+3x-4} = 1$

$(x+2)^{x^2+3x-4} = (x+2)^0$

$x^2+3x-4 = 0$

$(x+4)(x-1) = 0$

$x = -4,\ x = 1$

also $x+2=1$ or $x+2=-1$

$x=-1$ $\quad x=-3$

$\boxed{-4, -3, -1, 1}$

③ $xy - 2x = 12$

$x(y-2) = 12$

let $x = \{-12, \ldots, 12\}$

$-12(-1) = 12$ $\quad 1(12) = 12$

$-6(-2) = 12$ $\quad 2(6) = 12$

$-4(-3) = 12$ $\quad 3(4) = 12$

$-3(-4) = 12$ $\quad 4(3) = 12$

$-2(-6) = 12$ $\quad 6(2) = 12$

$-1(-12) = 12$ $\quad 12(1) = 12$

$\boxed{12}$ points

④ $f(z) = \frac{(z+2)(z-1)}{(z+i)(z-2)}$

$f(z) = \frac{z-1}{z-2}$

$f(3+2i) = \frac{3+2i-1}{3+2i-2}$

$= \frac{2+2i}{1+2i} \cdot \frac{1-2i}{1-2i}$

$= \frac{2-2i-4i^2}{1-4i^2}$

$= \frac{6-2i}{5}$

$= \boxed{\frac{6}{5} - \frac{2}{5}i}$

⑤ $(19^{20} \times 21^{22})(23^{24})(24^{25})$

$(19 \cdot 21 \cdot 23 \cdot 24)^{20}(21^2 \cdot 23^1 \cdot 24^5)$

21 ×21 → 1

23 ×23 → 9 ×23 → 7 ×23 → 1

24 ×24 → 6 ×24 → 4 ×24 → 6 ×24 → 4

$(1)(1)(4) = \boxed{4}$

⑥

$\log_2 k = \frac{1}{2}(\log_2 5) + 2\log_2 3 - 3$

$= \log_2 \sqrt{5} + \log_2 9 - \log_2 2^3$

$= \log_2(9\sqrt{5}) - \log_2 8$

$= \log_2\left(\frac{9\sqrt{5}}{8}\right)$

$k = \boxed{\frac{9\sqrt{5}}{8}}$

⑦ $x = \sin^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{1}{2}\right)$

$\sin x = \sin\left(\sin^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{1}{2}\right)\right)$

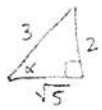
$= \sin\left(\sin^{-1}\left(\frac{2}{3}\right)\right)\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right) + \cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right)\sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

$= \left(\frac{2}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{5}}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{2}{6} + \frac{\sqrt{15}}{6}$

$= \boxed{\frac{2+\sqrt{15}}{6}}$

☺ $\sin(A+B) = \sin A\cos B + \cos A \sin B$



⑧ $8^{x+1} - 2^{(x+1)^2} = 0$

$2^{3(x+1)} - 2^{(x+1)^2} = 0$

$2^{3(x+1)} = 2^{(x+1)^2}$

$3x+3 = x^2+2x+1$

$0 = x^2 - x - 2$

$0 = (x+1)(x-2)$

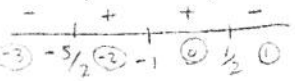
$x = -1,\ x = 2$

$\boxed{-1, 2}$

⑨ $3x(x+1)^2(2x-1) - 5(x+1)^3(2x-1) > 0$

$(2x-1)(x+1)^2(3x - 5(x+1)) > 0$

$(2x-1)(x+1)^2(-2x-5) > 0$



$\boxed{-2}$ as smallest integer in $(-5/2, -1) \cup (-1, 1/2)$

⑩ $\begin{matrix} 3 & 1 & x & 3 & 1 \\ 2 & 4 & 1 & 2 & 4 \\ 3 & 2 & y & 3 & 2 \end{matrix} = 2x + 5y - 3$

$-12x - 6 - 2y + 12y + 3 + 4x = 2x + 5y - 3$

$-8x + 10y - 3 = 2x + 5y - 3$

$5y = 10x$

$\frac{5}{10} = \frac{x}{y}$

$x : y = \boxed{1 : 2}$

(11) $\frac{7!\,4!}{2!} = \frac{n!}{3!}$

$\frac{7!\,4!\,3!}{2!} = n!$

$\frac{7!\,4\cdot3\cdot2\cdot3\cdot2!}{2!} = n!$

$7!\,8\cdot9 = n!$

$9! = n!$

$n = \boxed{9}$

(12) $(x-3)^2 + (y+1)^2 = 6^2$

center $\to (3,-1)$ } circle
radius $\to 6$

closer to Outside

$A = \pi(6)^2 - \pi(3)^2$

$A = \pi(6)^2$
$= 36\pi$

$= 36\pi - 9\pi$

$= 27\pi$

$\frac{27\pi}{36\pi} = \frac{27}{36} = \boxed{\frac{3}{4}}$

(13) $29! + 30! + 31!$

$29!\,(1 + 30 + 31)$

$29!\,(62)$

$29!\,(2\cdot31)$

$\boxed{31}$

(14) $4x^2 - 3x + 9 = 0$

$x = \frac{+3 \pm \sqrt{(-3)^2 - 4(4)(9)}}{2(4)}$

$x = \frac{3 \pm \sqrt{9 - 144}}{8}$

$x = \frac{3 \pm \sqrt{-135}}{8}$

$\left(\frac{3+\sqrt{-135}}{8}\right)^2 + \left(\frac{3-\sqrt{-135}}{8}\right)^2$

$\frac{9 + 6\sqrt{-135} - 135 + 9 - 6\sqrt{-135} - 135}{64}$

$\frac{-270 + 18}{64} = \frac{-252}{64} = \boxed{\frac{-63}{16}}$

(15) $3x + 4y = 12$

$y = \frac{-3x+12}{4}$

$y = -\frac{3x}{4} + 3$

	+6 or −6
$y = -\frac{3}{4}x + 9$	−3
$4y = -3x + 36$	−12

$3x + 4y = \boxed{36}$ $\quad 3x + 4y = \boxed{-12}$

(16) $f(x) = \frac{ax+3}{3x+c}$

V.A. $3x + c = 0$

$x = \frac{-c}{3}$

$4 = \frac{-c}{3}$

$c = -12$

H.A. $y = \frac{a}{3}$

$2 = \frac{a}{3}$

$6 = a$

$6 + (-12) = \boxed{-6}$

(17)

$9 \cdot 3^x = 3^{20}$

$3^2 \cdot 3^x = 3^{20}$

$3^{2+x} = 3^{20}$

$2 + x = 20$

$x = \boxed{18}$

(18) $6, \cdots, \cdots, \cdots, -18$

$a_5 = a_1 + d(5-1)$

$-18 = 6 + 4d$

$-24 = 4d$

$-6 = d$

$a_4 = 6 + -6(4-1)$

$= 6 - 6(3)$

$= 6 - 18$

$= -12$

$a_4 = a_1 r^{4-1}$

$-12 = a_1(-6)^3$

$\frac{-12}{(-6)^3} = a_1$

$a_3 = a_1 r^{3-1}$

$a_3 = \left(\frac{-12}{(-6)^3}\right)(-6)^2$

$= \frac{-12}{-6}$

$= \boxed{2}$

(19) $\frac{x+1}{2-x} \geq 2$

$\frac{x+1}{2-x} - 2 \geq 0$

$\frac{x+1}{2-x} - \frac{2(2-x)}{2-x} \geq 0$

$\frac{3x-3}{2-x} \geq 0$

$1 \leq x < 2$

$4 \leq 4x < 8$

integers 4, 5, 6, 7

$\boxed{4}$

(20) $\sum_{n=4}^{\infty} \left(\frac{3}{2}\right)^{10-3n} = \left(\frac{3}{2}\right)^{10-3(4)} + \left(\frac{3}{2}\right)^{10-3(5)} + \left(\frac{3}{2}\right)^{10-3(6)} + \cdots$

$= \left(\frac{3}{2}\right)^{-2} + \left(\frac{3}{2}\right)^{-5} + \left(\frac{3}{2}\right)^{-8} + \cdots$

$= \frac{4}{9} + \frac{32}{343} + \cdots$

$r = \frac{\frac{32}{343}}{\frac{4}{9}} = \frac{32}{343} \cdot \frac{9}{4} = \frac{8}{27}$

$S_\infty = \frac{a_1}{1-r}$

$= \frac{\frac{4}{9}}{1 - \frac{8}{27}} = \frac{\frac{4}{9}}{\frac{19}{27}} = \frac{4}{9} \cdot \frac{27}{19} = \frac{4 \cdot 3}{19} = \boxed{\frac{12}{19}}$