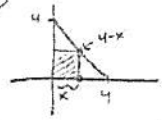
2000 RAA

(1)



$x=0$
$y=0$
$y=4-x$

<u>Calculus</u>

$A=x(4-x)$

$A=4x-x^2$

$A'=4-2x$

$0=4-2x$

$2=x$

$A=2(4-2)$

$=2(2)$

$=\boxed{4}$

<u>Non-Calculus</u>

$P=x+4-x+x+4-x$

$P=8$ perimeter is constant, largest area a square

so,

$8=x+y+x+y$

$8=2x+2y$

each side $=2$

$A=2(2)$

$A=\boxed{4}$

(2) roots: $1,\ \underbrace{1-i,\ 1+i}_{\text{complex pairs}}$

$(1)^3+a(1)^2+b(1)+c=0$

$1+a+b+c=0$

$a+b+c=\boxed{-1}$

(3) $p=.7$ (make) 3 out of 5 miss $=.3$

$P(3|5)={}_5C_3\ \underbrace{(.7)^3}_{\text{make } 3}\underbrace{(.3)^2}_{\text{miss } 2}$

$=\boxed{.3087}$

(4) $S_\infty=6$ S terms $=12$

$\frac{a}{1-r}=6$

$a=6(1-r)$

$\frac{a^2}{1-r^2}=12$

$\frac{[6(1-r)]^2}{1-r^2}=12$

$\frac{36(1-r)^2}{(1-r)(1+r)}=12$

$36-36r=12+12r$

$24=48r$

$\boxed{\frac{1}{2}=r}$

(5) $(\log_3 4)(\log_4 5)(\log_5 6)\ldots(\log_{31}(32))=\frac{1}{\log_{32}x}$

$\frac{\log 4}{\log 3}\cdot\frac{\log 5}{\log 4}\cdot\frac{\log 6}{\log 5}\ldots\frac{\log 32}{\log 31}=\frac{1}{\frac{\log x}{\log 32}}$

$\frac{\log 32}{\log 3}=\frac{\log 32}{\log x}$

$\boxed{x=3}$

(6) $a_3=a_2+a_1$ $a_1=1$

geometric $\rightarrow$ $r=\frac{a_2}{a_1}=a_2$, so $r=a_2$

$a_3=a_1r^2$

$a_3=r^2$

$a_3=a_2+a_1$

$r^2=r+1$

$r^2-r-1=0$

$r=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}$

$r=\frac{1\pm\sqrt5}{2}$

$r=\frac{1+\sqrt5}{2}$

$a_7=a_1r^6$

$=1(r^2)^3$

$=1(r+1)^3$

$=\left(\frac{1+\sqrt5}{2}+1\right)^3$

$=\left(\frac{3+\sqrt5}{2}\right)^3$

$=\left(\frac{3+\sqrt5}{2}\right)\left(\frac{3+\sqrt5}{2}\right)\left(\frac{3+\sqrt5}{2}\right)$

$=\left(\frac{9+6\sqrt5+5}{4}\right)\left(\frac{3+\sqrt5}{2}\right)$

$=\frac{(14+6\sqrt5)(3+\sqrt5)}{8}$

$=\frac{42+18\sqrt5+14\sqrt5+30}{8}=\frac{72+32\sqrt5}{8}=\boxed{9+4\sqrt5}$

(7) $x+by+cz+d=0$

$2+3b+5c+d=0$ $(2,3,5)$

$-1+6b+8c+d=0$ $(-1,6,8)$

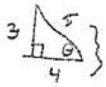
$2b+c+d=0$ $(0,2,1)$

$[A]=\left[\begin{array}{ccc|c}3&5&1&-2\\6&8&1&1\\2&1&1&0\end{array}\right]$ $\text{rref}[A]\Rightarrow \begin{array}{ccc} b & c & d\\ {[2,} & -1, & -3] \end{array}$

$\boxed{x+2y-z-3=0}$

(8) $\sin(2 \underbrace{\arccos \tfrac{4}{5}}) = \boxed{\frac{24}{25}}$ by calculator

$\downarrow$ by hand



$\} \arccos \frac{4}{5} = \theta$

$= \sin 2\theta$

$= 2 \sin\theta \cos\theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \boxed{\frac{24}{25}}$

(9) $A = Pe^{rt}$

18 months = 1.5 years
1500 = principal
2300 = end Amount 1.5 yrs.
3300 = 2nd principal

$2300 = 1500e^{r(1.5)}$

$\frac{2300}{1500} = e^{1.5r}$

$r = \frac{\ln\left(\frac{2300}{1500}\right)}{1.5}$

$r \approx .28496$ store in calc

$A = 3300e^{r(2)}$

$A = 5834.8457$

$= \boxed{\$5835}$

(10) $y = ax^2 + bx + c$

$9 = a(1)^2 + b(1) + c$

$6 = a(4)^2 + b(4) + c$

$14 = a(6)^2 + b(6) + c$

$[A] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 16 & 4 & 1 & 6 \\ 36 & 6 & 1 & 14 \end{array}\right] \rightarrow \text{rref}[A] = (1, -6, 14)$

$\boxed{y = x^2 - 6x + 14}$

(11) let 1st die be any # → say 2 ... probability is 6/6

then, 2nd die has 5/6 chance of not being at 2

and say 2nd die is a 1.

then, 3rd die has 4/6 change of not being at 2 or 1.

and say 3rd die is a 6.

Then, 4th die has 3/6 chance of not being a 2 or 1 or 6.

so.... $\underset{\text{1st die}}{\frac{6}{6}} \cdot \underset{\text{2nd die}}{\frac{5}{6}} \cdot \underset{\text{3rd die}}{\frac{4}{6}} \cdot \underset{\text{4th die}}{\frac{3}{6}} = \boxed{\frac{5}{18}}$

(12) $2\log(x-2) - \log(x-3) = 1$

$\log(x-2)^2 - \log(x-3) = 1$

${}_{10}\log\left(\frac{(x-2)^2}{x-3}\right) = {}_{10}1$

$\frac{(x-2)^2}{x-3} = 10^1$

$(x-2)^2 = 10x - 30$

$x^2 - 4x + 4 = 10x - 30$

$x^2 - 14x + 34 = 0$

QUAD FORM PROGRAM $\nearrow$ $x = \frac{14 \pm \sqrt{14^2 - 4(1)(34)}}{2}$

$x = \frac{14 \pm \sqrt{60}}{2}$

$x = \frac{14 \pm 2\sqrt{15}}{2}$

$x = \boxed{7 + \sqrt{15}}$

(13) distance pt to line $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$(x_1, y_1) = (1, 5)$

$ax + by + c = 0 \Rightarrow 2x - 3y - 7 = 0$

$= \frac{|2(1) - 3(5) - 7|}{\sqrt{2^2 + (-3)^2}}$

$= \frac{|-20|}{\sqrt{13}}$

$= \boxed{\frac{20}{\sqrt{13}}}$ or $\frac{20\sqrt{13}}{13}$

(14) $\sec^2\theta = 3\sec\theta - 2$ $\quad 0 \le \theta < 2\pi$

$\sec^2\theta - 3\sec\theta + 2 = 0$

$(\sec\theta - 2)(\sec\theta - 1) = 0$

$\sec\theta = 2 \qquad \sec\theta = 1$

$\frac{1}{\cos\theta} = 2 \qquad \frac{1}{\cos\theta} = 1$

$\cos\theta = \frac{1}{2} \qquad \cos\theta = 1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \qquad \theta = 0,$

$\boxed{0, \frac{\pi}{3}, \frac{5\pi}{3}}$

2000 RAA

(15) $\begin{vmatrix} 1 & 3 & 2 \\ 4 & t & 1 \\ 2 & 2 & t \end{vmatrix} = -43$

$\begin{matrix} 1 & 3 & 2 & 1 & 3 \\ 4 & t & 1 & 4 & t \\ 2 & 2 & t & 2 & 2 \end{matrix}$

$-4t - 2 - 12t \qquad t^2 + 6 + 16$

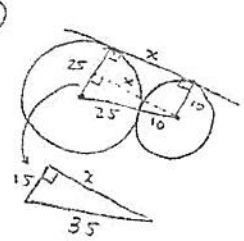
$t^2 - 16t + 20 = -43$

$t^2 - 16t + 63 = 0$

$(t-7)(t-9) = 0$

$\boxed{t = 7, \ t = 9}$

(16)



$15^2 + x^2 = 35^2$

$x^2 = 1000$

$x = \sqrt{1000}$

$x = \boxed{10\sqrt{10}}$

(17) $200\left(1 + \frac{.06}{12}\right)^{180}$

$+ 200\left(1 + \frac{.06}{12}\right)^{179}$

$+ 200\left(1 + \frac{.06}{12}\right)^{178}$

$\vdots$

$+ 200\left(1 + \frac{.06}{12}\right)^{1}$

$\sum_{k=1}^{180} 200\left(1 + \frac{.06}{12}\right)^{k} = \boxed{\$58\,454.56}$

(18) $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots \frac{1}{(2n-1)(2n+1)} + \cdots =$

$\frac{1}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1}$ partial fractions

$1 = A(2n+1) + B(2n-1)$

$1 = 2An + A + 2Bn - B$

$2A + 2B = 0 \qquad A - B = 1$

$2A - 2B = 2 \qquad \frac{1}{2} - B = 1$

$4A = 2 \qquad B = -\frac{1}{2}$

$A = \frac{1}{2}$

$\frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\left(\frac{1}{2n-1}\right) - \frac{1}{2}\left(\frac{1}{2n+1}\right)$

$= \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$

sum of 1st 3 terms

$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7}\right]$

$= \frac{1}{2}\left[\frac{1}{1} - \frac{1}{7}\right]$

$= \frac{1}{2}\left[\frac{1}{1} - \frac{1}{2\cdot 3 + 1}\right]$

So, sum of 1st 12345 terms

$= \frac{1}{2}\left[\frac{1}{1} - \frac{1}{2(12345)+1}\right]$

$= \frac{1}{2}\left[\frac{1}{1} - \frac{1}{24691}\right]$

$= \frac{1}{2}\left[\frac{24690}{24691}\right]$

$= \boxed{\frac{12345}{24691}}$

(19) $(2,3)$ on $y = f(x+1)$, let $y = x+1$ & check pt.

$x + 1 = f(x+1) \qquad 3 = 2 + 1$ ✓

So, $x = f(x)$

and $f^{-1}(x) = x$ $\Big\}\ f^{-1}(x) = f(x)$

$-f^{-1}(x) = -x$

$-f^{-1}(3) = -3$

$\boxed{(3, -3)}$

(20) $x^2 - 4x + y^2 + 8y - 15605 = 0$

$x^2 - 4x + 4 + y^2 + 8y + 16 = 15605 + 4 + 16$

$(x-2)^2 + (y+4)^2 = 15625$

$(x-2)^2 + (y+4)^2 = 125^2$

$r_1 = 125, \ r_2 = \frac{3}{5}(125) \ \cdots \ r_4 = \left(\frac{3}{5}\right)^3(125)$

$r_4 = 27$

$A = \pi(27)^2$

$= \boxed{729\pi}$