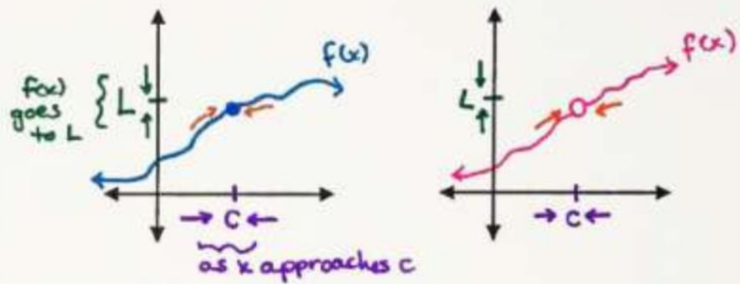
Summary of Limits (Learned in PreCalculus)

What is a Limit?

$$\lim_{x \to c} f(x) = L$$

What does $\lim\limits_{x \to c} f(x) = L$ mean?

As x approaches c (from either side), then $f(x)$ becomes close to L.



- ❋ By making x get very close to c, $f(x)$ gets very close to L.
- ❋ As the input, x, approaches some value c, then the function, $f(x)$, approaches the value of L.

$\lim\limits_{x \to c} f(x) = L$ provided that these 3 conditions are true:

$$\begin{cases} \text{i. } \lim\limits_{x \to c^-} f(x) \text{ exists} \\ \text{ii. } \lim\limits_{x \to c^+} f(x) \text{ exists} \\ \text{iii. } \lim\limits_{x \to c^+} f(x) = \lim\limits_{x \to c^-} f(x) \end{cases}$$

The "lim" tells us we're looking for a limit value, not a function value.

This tells us which function we're working with.

$$\lim_{x \to 6} f(x) = 4$$

This tells us what the variable is, and what it is approaching.

This is the value the function is approaching.

Ways to Evaluate a Limit

Numerically – use/make a table of values

https://www.mathkanection.com/uploads/8/4/4/3/84436602/10.3.4_limits_numerically_guided_notes_filled_in_2016.pdf

Analytically – use algebra

- Replace value of c for x (if possible)
- Result is indeterminate, $\frac{0}{0}$? Then, use algebraic methods (factor & reduce, multiply by the conjugate, use trig limits, etc)

https://www.mathkanection.com/uploads/8/4/4/3/84436602/10.3.2_limits_analytically_guided_notes_filled_in_2016.pdf

Graphically – use a graph

https://www.mathkanection.com/uploads/8/4/4/3/84436602/10.3.3_limits_graphically_guided_notes_filled_in_2016.pdf

Infinite Limits and Limits at Infinity

Infinite Limit – a limit in which $f(x)$ increases or decreases without bound as $x \to c$.

– Vertical Asymptotes (after reducing, denominator = 0)

If $\lim_{x \to c^-} f(x) = \infty \; or - \infty$ or $\lim_{x \to c^+} f(x) = \infty \; or - \infty$, then $f(x)$ has a vertical asymptote at $x = c$.

https://www.mathkanection.com/uploads/8/4/4/3/84436602/1.2.4_va_ha_guided_notes_filled_in_2016.pdf

Limits at Infinity – end behavior of a graph/function

– Horizontal Asymptote at $y = L$ if $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$

Conclusion:
If degree of Numerator = degree of Denominator, then $\lim_{x \to \infty} f(x) =$ coefficients of leading terms of Numerator/Denominator
(and, Horizontal Asymptote @ $y =$ coefficients (#))

Conclusion:
If degree of Numerator < degree of Denominator, then $\lim_{x \to \infty} f(x) = 0$
(and, Horizontal Asymptote @ $y = 0$

Conclusion:
If degree of Numerator > degree of Denominator, then $\lim_{x \to \infty} f(x) = +\infty$ or $-\infty$
(NO Horizontal Asymptote)

https://www.mathkanection.com/uploads/8/4/4/3/84436602/10.3.1_limits_at_infinity_guided_notes_filled_in_2016.pdf

Properties of Limits

Let c be a real number and let f and g be functions with limits that exist, then:

Sum or Difference: $\lim_{x \to c}(f(x) \pm g(x)) = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$

Product: $\lim_{x \to c}(f(x) \cdot g(x)) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$

Coefficient: $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)$

Quotient: $\lim_{x \to c} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$

Power or Root: $\lim_{x \to c}(f(x))^n = \left(\lim_{x \to c} f(x)\right)^n$

$\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$

Composite: $\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right)$